Important Concepts . . .

Preview Review

Mathematics Grade 9

W1 - Lesson 3: Rational Numbers

Important Concepts of Grade 9 Mathematics

Materials Required

Paper
Pencil
Calculator

No Textbook Required

This is a stand-alone course.

Mathematics Grade 9
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Preview/Review W1 - Lesson 3
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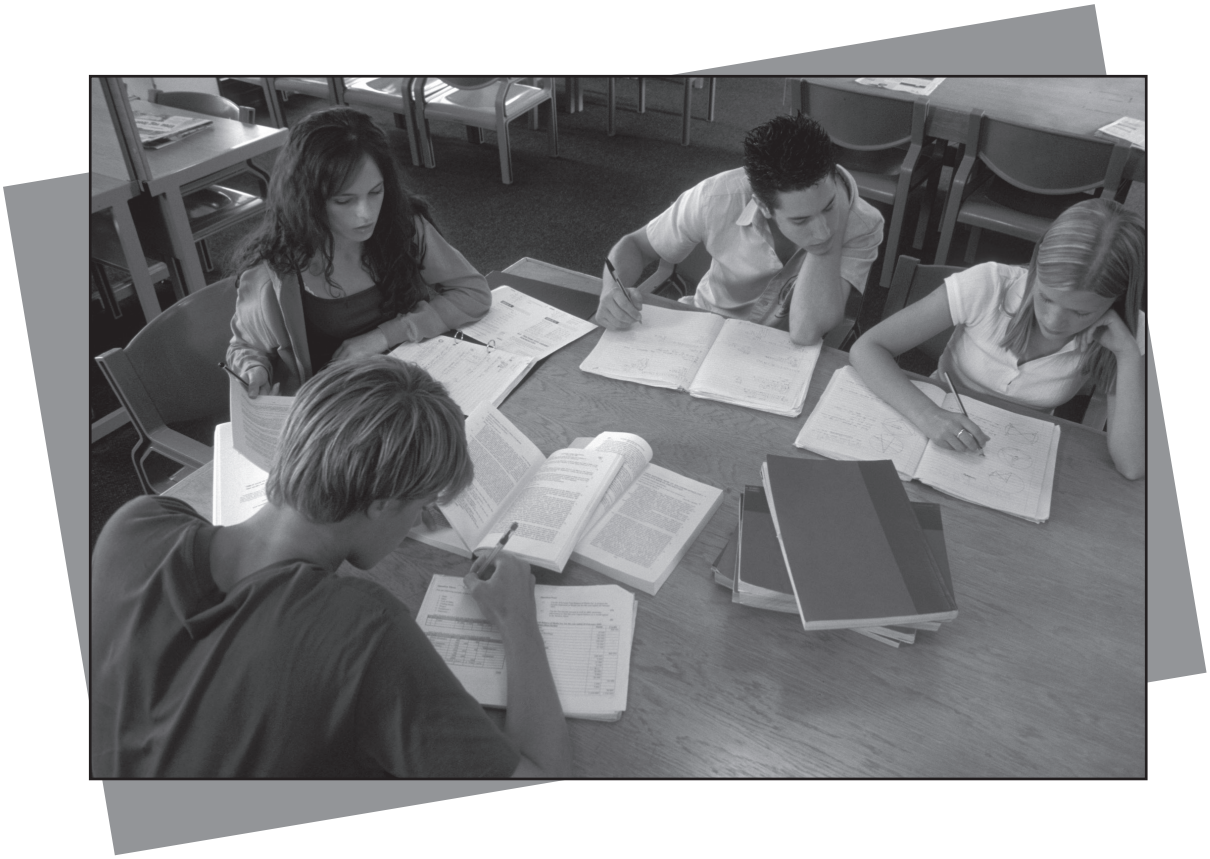
Preview/Review Concepts for Grade Nine Mathematics



W1 – Lesson 3:

Rational Numbers

OBJECTIVES

By the end of this lesson, you will be able to:

- Describe and give examples of rational numbers.
- Compare and order rational numbers.
- Identify a rational number that is between two given rational numbers.
- Order rational numbers on a number line.

GLOSSARY

Rational Number: Any number that can be written as a ratio of two integers. It can be positive, negative, a fraction or a decimal as long as it can be written as a fraction where the numerator and denominator are both integers.

Ex. $\frac{1}{2}$, 0.75 (can be written as $\frac{3}{4}$), –1 (can be written as $-\frac{1}{1}$), 0.988 (can be written as $\frac{988}{1000}$) . . .

Fraction: Part of a whole. A number written with the denominator indicating how many parts the whole is divided into, and the numerator indicating how many there are.

Ex. $\frac{3}{4}$ <– – numerator / <– – denominator, $-\frac{5}{2}$, $1\frac{4}{9}$

Repeating Decimal: A decimal number that has digits that repeat forever.

Ex. 0.333 . . . (the 3 repeats forever), 0.128222 . . . (the 2 repeats forever)

The part that repeats is usually shown by placing dots over the first and last digits of the repeating pattern, or sometimes a line over the pattern: $0.\dot{3}$ or $0.\overline{3}$

Integer: A number with no fractional part. This includes the counting numbers {1, 2, 3, . . .}, zero {0}, and the negative of the counting numbers {–1, –2, –3, . . .}

Ex. –16, –3, 0, 1, 198

W1 – Lesson 3: Rational Numbers

Materials required:

- Paper, Pencil, and Calculator

Part 1: Describing Rational Numbers

A **rational number** is any number that can be written as a ratio of two integers. In other words, a number is rational if it can be written as a fraction where the numerator and denominator are both integers.

The term "rational" comes from the word "ratio," because the rational numbers are the ones that can be written in the ratio form $\frac{p}{q}$ where p and q are integers.

Example 1 – Fractions

A fraction is a ratio where the bottom part of the fraction (the denominator) indicates how many parts the whole is divided into, and the top part of the fraction (the numerator) indicates how many there are in total. It is written in the ratio form $\frac{p}{q}$ where p and q are integers and $q \neq 0$. A fraction is an example of one type of rational number. Some examples of fractions include: $\frac{5}{6}, \frac{7}{2}, 4\frac{3}{4}, -\frac{3}{5}$.

Example 2 – Integers

Every integer is a rational number. This is because every integer, otherwise known as "n" can be expressed as the fraction $\frac{n}{1}$.

For example $5 = \frac{5}{1}$ and thus 5 is a rational number.

More examples of integers include: $-6 = -\frac{6}{1}, 324 = \frac{324}{1}$

Example 3 – Decimals

All terminating and repeating decimals are rational numbers. A decimal is also part of a whole. A decimal is always calculated by using a fraction. To calculate the decimal equivalent of a fraction, take the numerator and divide it by the denominator of a fraction.

$\frac{3}{4} = 3 \div 4 = 0.75$ This is called a **terminating decimal**.

Sometimes the result is a **repeating decimal** . . . the decimal goes on forever.

$$-\frac{2}{3} = -2 \div 3 = -0.6666666\ldots, \text{ or } 0.\overline{6} \text{ or } -0.\dot{6}$$

The line or the dot over the 6 indicates that the 6 is repeating. Therefore, always place a line or a dot over top of the number(s) that repeat.

Converting Decimals to Fractions

To convert a decimal back to a fraction, place the decimal over 10, 100, or 1000 . . . etc.

0.4 would become $\frac{4}{10}$

0.44 would become $\frac{44}{100}$

0.444 would become $\frac{444}{1000}$

Then reduce to lowest terms:

$\frac{4}{10}$ would become $\frac{2}{5}$

$\frac{44}{100}$ would become $\frac{11}{25}$

$\frac{444}{1000}$ would become $\frac{111}{250}$

To convert a repeating decimal back to a fraction, simply place the decimal over 9, 99, or 999 . . . etc, then reduce the fraction.

0.333… would become $\frac{333}{999}$. Then reduce to lowest terms. $\frac{333}{999} = \frac{1}{3}$

Practice Questions

1. Express the following rational numbers as a fraction.

   a. 5

   _____

   b. –12

   _____

2. Write each decimal as a fraction. Be sure to express the fraction in reduced form.

   a. 0.7

   _____

   b. –0.15

   _____

   c. 0.432

   _____

   d. $-0.0\overline{66}$

   _____

3. Write each fraction as a decimal.

   a. $\frac{8}{10}$

   _____

   b. $\frac{-725}{1000}$

   _____

   c. $-\frac{8}{16}$

   _____

   d. $\frac{12}{60}$

   _____

   e. $\frac{14}{84}$

   _____

   f. $2\frac{3}{5}$

   _____

Part 2: Compare Rational Numbers

To compare rational numbers, apply rules to compare equivalent rational numbers.

Example 1

Which fraction is greater?

$$-\frac{1}{3} \text{ or } -\frac{8}{18}$$

To compare these fractions, express them as equivalent fractions with a common denominator.

To make equivalent fractions, either multiply or divide both the numerator and the denominator by the same number.

$$-\frac{1}{3} \xrightarrow[\times 6]{\times 6} -\frac{6}{18}$$

A common denominator of these two fractions is 18.

When the denominators are the same, compare the numerators.

$-\frac{6}{18} > -\frac{8}{18}$ because –6 > –8. Remember, on a number line –6 is closer to zero than –8, therefore it is the larger value.

Example 2

Using decimals to compare rational numbers is also possible. Using the above example, the fractions can be written as decimal numbers.

$-\frac{1}{3} = -0.333\ldots$ and $-\frac{8}{18} = -0.444\ldots$

$-0.333\ldots > -0.444\ldots$

Therefore $-\frac{1}{3}$ is the greater fraction.

Practice Questions

1. Write each rational number as an equivalent fraction.

   a. $\frac{5}{4}$ _____

   b. $\frac{7}{-2}$ _____

   c. $-\frac{9}{12}$ _____

2. Which value in the pair is greater?

   a. $\frac{4}{7}$ $\frac{2}{3}$

   b. $\frac{7}{10}$ $\frac{3}{5}$

   c. $\frac{-4}{3}$ $\frac{-5}{3}$

3. Which value in the set is the smallest?

   a. $\frac{7}{4}$ $\frac{6}{5}$ $\frac{10}{8}$

   b. $\frac{-8}{5}$ $\frac{12}{-10}$ $\frac{-5}{3}$

Part 3: Identifying Rational Numbers Between Two Given Rational Numbers

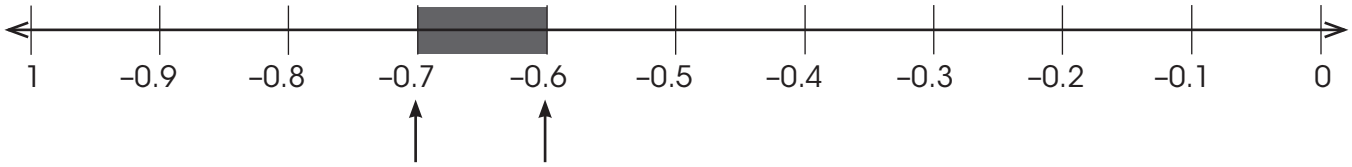
Rational numbers can be positive, negative or zero. They include integers, positive and negative fractions, mixed numbers and decimal numbers.

Using a number line is a strategy to identify a rational number between two given rational numbers.

On a horizontal number line, a larger rational number is to the right of a smaller rational number. Opposite rational numbers are the same distance in opposite directions from zero.

Example 1

Identify a fraction and a decimal between –0.6 and –0.7.



Any decimal in the highlighted zone is a decimal between –0.6 and –0.7.

Examples are: –0.61, –0.62, –0.63, –0.64, –0.65, –0.66, –0.67. –0.68, –0.69

Choose a decimal and convert to a fraction:

–0.67 becomes $-\frac{67}{100}$

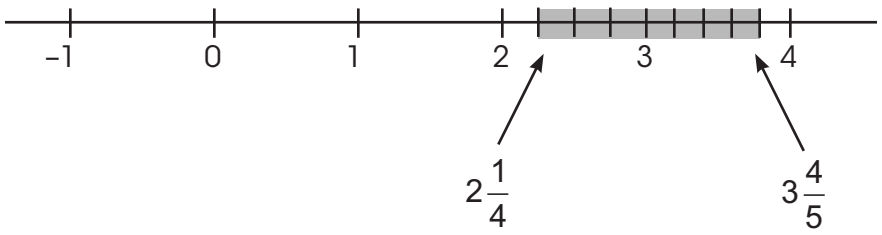
Therefore,

One decimal between –0.6 and –0.7 is –0.65.

One fraction between –0.6 and –0.7 is $-\frac{67}{100}$.

Example 2

Identify a fraction and a decimal between $2\frac{1}{4}$ and $3\frac{4}{5}$.



Any fraction in the highlighted zone is a fraction between $2\frac{1}{3}$ and $3\frac{4}{5}$.

Examples are: $2\frac{2}{3}, 3\frac{1}{5}, 3\frac{2}{5}, 3\frac{3}{5}$

Take the given fractions and convert to decimals

$2\frac{1}{4}$ becomes 2.25

$3\frac{4}{5}$ becomes 3.8

Any decimal between 2.25 and 3.8 is between the fraction $2\frac{1}{3}$ and $3\frac{4}{5}$.

Examples are: 2.26, 2.27, 2.28, 2.29, 2.30 . . . 3.2, 3.3, 3.4, 3.5, 3.6, 3.7

Therefore,

One fraction between $2\frac{1}{3}$ and $3\frac{4}{5}$ is $3\frac{1}{5}$,

One decimal between $2\frac{1}{3}$ and $3\frac{4}{5}$ is 2.29.

Practice Questions

1. What is the opposite of each rational number?

   a. –4.23 b. $\frac{4}{5}$ c. $-\frac{16}{5}$

   _____ _____ _____

2. Identify a decimal number between each pair of rational numbers.

   a. $\frac{3}{5}, \frac{4}{5}$ b. $-\frac{5}{6}, 1$ c. $\frac{3}{2}, \frac{17}{10}$

   _____ _____ _____

3. Identify a fraction between each pair of rational numbers.

   a. 0.2, 0.3 b. –0.74, –0.76 c. 0, –0.1

   _____ _____ _____

Part 4: Comparing and Ordering Rational Numbers

To order a set of rational numbers in which some are fractions or integers and some are decimals, convert them all to decimals. Then line up the numbers on the decimal point, use trailing zeros to fill out each number to the same number of decimal digits, and then compare.

Example 1

Place in order from least to greatest and place them on a number line.

$0.41, -0.33, 0.95, -\frac{35}{50}, 2\frac{1}{2}$ and $-1.2.$

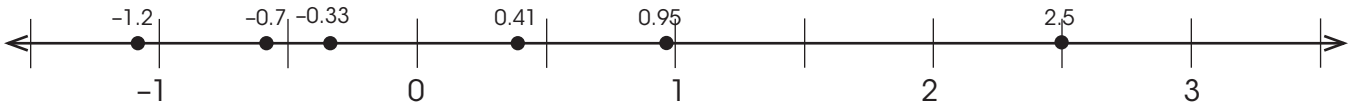
Step 1: Convert all fractions to decimals to compare.

$-\frac{35}{50} = -0.7 \qquad 2\frac{1}{2} - 2.5$

Step 2: Order the decimals:

$-1.2 < -0.7 < -0.33 < 0.41 < 0.95 < 2.5$

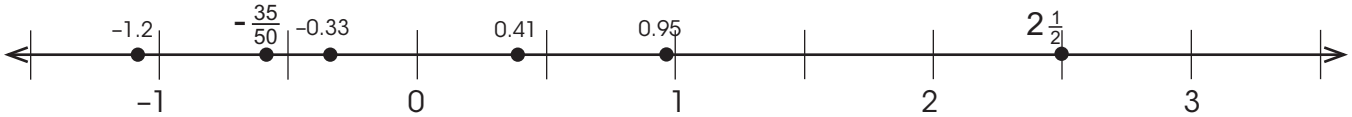
Step 3: Use the converted decimals to place the dots on a number line:



Step 4: Put the converted decimals back to fractional form.

$-1.2 < -\frac{35}{50} < -0.33 < 0.41 < 0.95 < 2\frac{1}{2}$

Step 5: Use the original numbers given to label the dots on a number line.



Example 2

Place the correct sign, >, <, = to make the below statements true.

$-4.25 \qquad -\frac{27}{4}$

Convert the fraction to a decimal $-\frac{27}{4} = -6.75$.

Therefore, $-4.25 \quad > \quad -\frac{27}{4}$

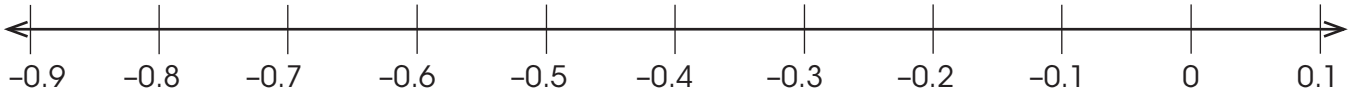
Practice Questions

1. Order each set from least to greatest.

$$\frac{7}{13}, -0.666\ldots \frac{1}{3}, 0.5, -\frac{55}{99}$$

2. Order each set from least to greatest and place them on the number line.

a. $-0.1, \frac{1}{10}, -0.\overline{7}\ldots, -\frac{7}{10}, -\frac{4}{7}$



3. Which rational number is greater?

a. $4.25 \qquad \frac{9}{50}$

b. $2\frac{2}{5} \qquad \frac{15}{7}$

c. $-6.11 \qquad -\frac{29}{4}$

d. $3\frac{3}{7} \qquad 3.222\ldots$

Lesson 3 Assignment

1. Identify a decimal number between each pair of rational numbers.

   a. $\frac{32}{36}, \frac{24}{48}$

   _____

   b. $-\frac{5}{2}, -\frac{7}{3}$

   _____

2. What is the opposite of each rational number?

   a. $-2\frac{4}{5}$

   _____

   b. $5\frac{3}{5}$

   _____

   c. $\frac{3}{7}$

   _____

3. Identify a fraction between each pair of rational numbers. Answers may vary.

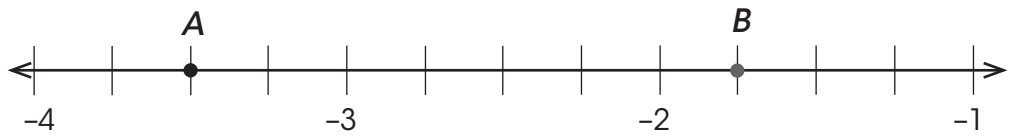
   a. 0.375, 0.38

   _____

   b. –0.6565656 . . . , –0.7

   _____

4. Which values describe the positions of *A* and *B*? Put your answer in both decimal and fractional form.



_____

_____

5. Place <, > or = to make each number sentence true.

a. $-0.666\ldots$ _____ $-\frac{3}{5}$

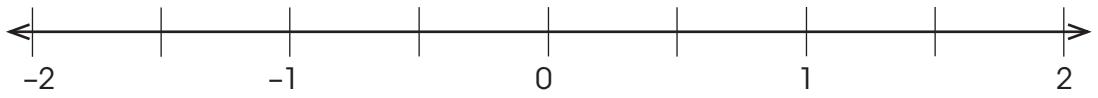
b. $\frac{2}{21}$ _____ $0.222\ldots$

c. $5\frac{5}{9}$ _____ $5.4545\ldots$

d. -4.066 _____ $-\frac{15}{4}$

6. Place the following in ascending order, and then place them on the given number line.

$\frac{5}{7}, -1.4, \frac{-7}{-5}, \frac{-21}{20}, -1\frac{3}{5}, \frac{-28}{-14}$

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