Important Concepts . . .

Preview Review

Mathematics Grade 9 TEACHER KEY

W2 - Lesson 7: Solving Linear Relations

Important Concepts of Grade 9 Mathematics

Materials Required

Paper
Pencil
Graph Paper
Calculator

No Textbook Required

This is a stand-alone course.

Mathematics Grade 9
Version 6
Preview/Review W2 - Lesson 7
ISBN: 978-1-927090-00-8

Publisher: Alberta Distance Learning Centre
Written by: Lynee Fyfe
Reviewed by: Danielle Winter

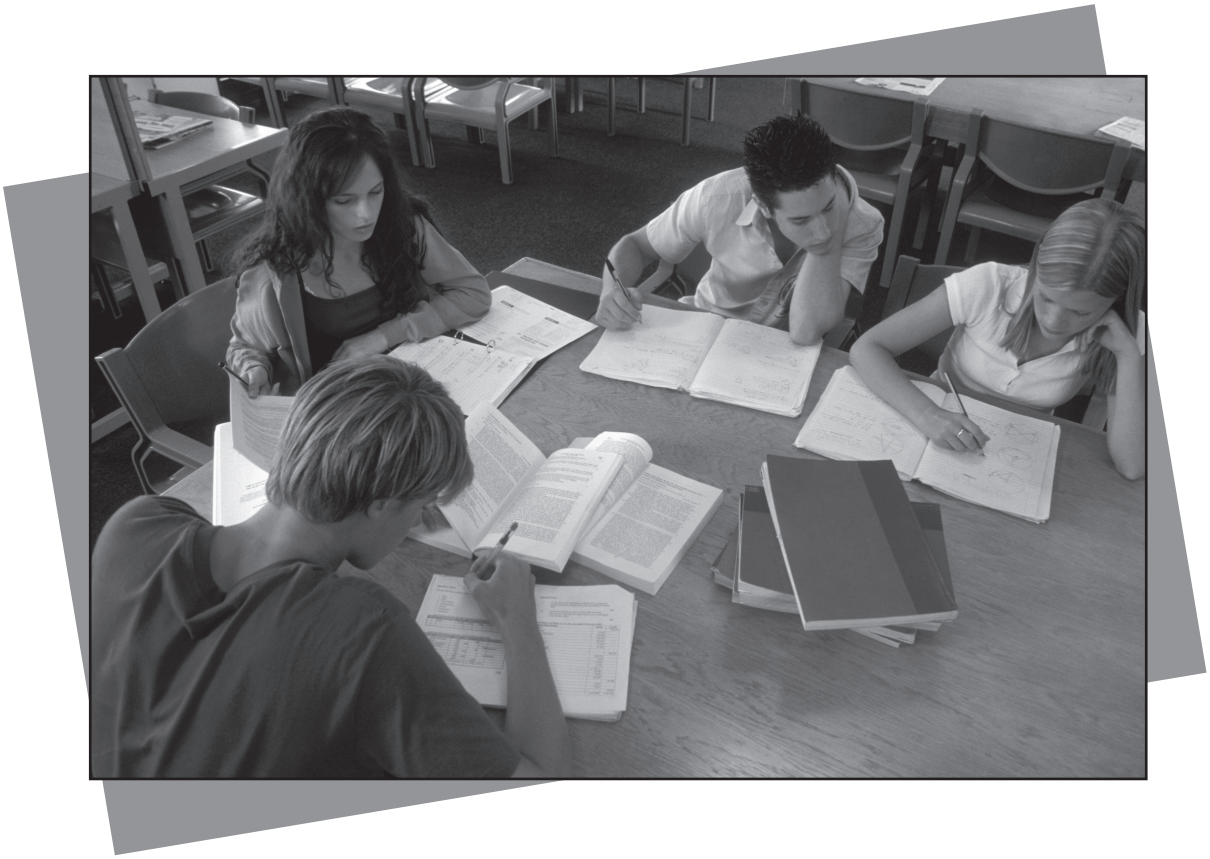
Project Coordinator: Danielle Winter
Preview/Review Publishing Coordinating Team: Julie Reschke

Alberta Distance Learning Centre has an Internet site that you may find useful. The address is as follows: **http://www.adlc.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Preview/Review Concepts for Grade Nine Mathematics

Teacher Key



W2 – Lesson 7:

Solving Linear Relations

OBJECTIVES

By the end of this lesson, you will be able to:

- Model the solution of a given linear equation, using concrete or pictorial representations, and record the process.
- Verify by substitution whether a given rational number is a solution to a given linear equation.
- Solve a given linear equation symbolically.
- Solve a given problem, using a linear equation, and record the process.

GLOSSARY

Algebraic equation: A number sentence containing a variable and an = sign. For example:

$3x + 4 = 7$
or
$2a = 6$

Algebraic expression: Similar to an equation, but does not contain an = sign. For example:

$3a + 2$
or
$2x + 37y + 7$

Linear equation: An equation of the form $y = ax + b$. The graph forms a non-vertical and non-horizontal straight line when graphed.

Linear relation: Relationships between two variables (usually x and y) that form a straight non-vertical and non-horizontal line when graphed.

Ordered pair: A related pair of values that correspond to a point on a graph; an ordered pair is written in the form (x, y).

Table of Values: A series of numbers used to substitute one variable within an equation in order to determine the value of the other (unknown) variable.

W2 – Lesson 7: Solving Linear Relations

Materials required:

- Paper, Pencil, Graph Paper, and Calculator

Part 1: Solving Equations *ax = b, x/a = b, a/x = b*

An equation is a mathematical statement containing a variable and an equal sign. Some examples of equations are:

$3x = -4$ $\qquad \frac{y}{2} = 1$ $\qquad \frac{8}{z} = 6$

In the equation $2.4f + 3.6 = -7.8$

- f is the variable. This represents an unknown number.
- 2.4 is the numerical coefficient.
- 3.6 and –7.8 are the constants.

Example 1 – Solving Equations with a Fraction

To solve for the variable x, use the opposite operation. The opposite operation "undoes" another operation. Examples of opposite operations are:

- Addition and subtraction
- Multiplication and division

$$2x = \frac{3}{4}$$

$4(2x) = \frac{3}{4}(4)$ Multiplying both sides by 4 removes the denominator.

$\frac{8x}{3} = \frac{3}{8}$ Divide both sides by 8 isolates the variable.

$$x = \frac{3}{8}$$

Example 2 – Solving Equations with 2 Fractions

When both sides are fractions, solve by cross multiplying.

$$\frac{y}{25} = \frac{4}{5}$$

$$\frac{y}{25} \times \frac{4}{5}$$

$$4(25) = 5(y)$$

$$100 = 5y$$

$$\frac{100}{5} = \frac{5y}{5}$$

$$20 = y$$

Recall – To verify an answer, simply substitute the solution in the equation, then check to make sure the right side = the left side.

Verification

$$\frac{y}{25} = \frac{4}{5}$$

$$\frac{20}{25} = \frac{4}{5}$$

Left Side = Right Side

Example 3 – Solving Equations with Decimals

$$\frac{t}{0.28} = -4.5$$

To solve, isolate the variable by applying the opposite operation.

$$\cancel{0.28} \times \frac{t}{\cancel{0.28}} = -4.5 \times 0.28$$

$$t = -1.26$$

Recall – when multiplying a negative by a positive, the product is negative.

Example 4 – Applying Equations

Kathy is buying a pair of shoes. The store is having a sale of 25% off the regular price. The shoes are on sale for $176.25, what is the regular price of the shoes? To solve this problem, create an equation to represent the situation.

Let p represent the regular price of the shoes.

The sale price is 75% of the regular price. So the sale price is $0.75p$.

Since the sale price is $176.25, an equation that represents this is

$$0.75p = 176.25$$

$$0.75 \mid \frac{0.75}{p} = \frac{176.25}{0.75}$$

$$p = 235$$

So the regular price of the shoes is $235.00.

Practice Questions

1. Solve. Verify your solutions for parts a and b.

a. $3 = \frac{x}{4}$

$\boldsymbol{x = 12}$

Verification

$$\boldsymbol{3 = \frac{x}{4}}$$
$$\boldsymbol{3 = \frac{12}{4}}$$
$$\boldsymbol{3 = 3}$$

b. $\frac{h}{4.1} = 3.6$

$\boldsymbol{h = 14.76}$

Verification

$$\boldsymbol{\frac{h}{4.1} = 3.6}$$
$$\boldsymbol{\frac{14.76}{4} = 3.6}$$
$$\boldsymbol{3.6 = 3.6}$$

c. $-\frac{7}{6} = -\frac{4}{3n}$

$$\boldsymbol{21n = 24}$$
$$\boldsymbol{n = \frac{24}{21}}$$
$$\boldsymbol{= \frac{8}{7}}$$
$$\boldsymbol{= 1\frac{1}{7}}$$

d. $-5.5 = \frac{1.1}{a}$

$$\boldsymbol{-5.5a = 1.1}$$
$$\boldsymbol{a = -\frac{1.1}{5.5}}$$
$$\boldsymbol{a = -0.2}$$

2. Kathy scored 17 correct on her math test. She received a mark of 68%. How many marks in total were on the test? To solve this problem, create an equation to represent the situation.

$$\frac{17}{x} = \frac{68}{100}$$
$$1700 = 68x$$
$$x = 25$$

Kathy's math test had 25 marks in total.

3. The diameter, d, of a circle is related to the circumference, C, by the formula $\frac{C}{d} = \pi$. Calculate the diameter of a circle with a circumference of 54.5 cm. Round your answer to the nearest tenth.

$$\frac{c}{d} = \pi$$
$$\frac{54.5}{c} = \pi$$

d = 17.4 if 3.14 was used, or 17.3 if π was used.

Part 2: Solving Equations $ax + b = c$, $\frac{x}{a} + b = c$

Linear equations are solved by isolating the variables. Two-step equations will require multiple processes to solve for the variable. Use the inverse operations to solve for a variable.

- Addition and subtraction are inverse operations.
- Multiplication and division are inverse operations.

Example 1 – Solving Two Step Equations with Fractions

Solve.

$$2x + \frac{1}{10} = \frac{3}{5}$$

$$(10)2x + (10)\ \frac{1}{10} = \frac{3}{5}\ (10)$$

$$20x + 1 = 6 \quad \text{Subtract 1 from both sides.}$$

$$20x = 5$$

$$20x \mid 5 = 5 \mid 5$$

$$4x = 1$$

$$4x \mid 4 = 1 \mid 4$$

$$x = \frac{1}{4}$$

To remove the fractions, multiply each term by the lowest common denominator (LCD).

To isolate the variable, use the reverse order of operations. Add or subtract first, then multiply or divide.

Example 2 – Solving Two Step Equations with Decimals

Solve.

$$\frac{r}{2.8} - 2.5 = -3.7$$

$$\frac{r}{2.8} - 2.5 + 2.5 = -3.7 + 2.5$$

$$\frac{r}{2.8} = -1.2$$

$$\cancel{2.8} \times \frac{r}{\cancel{2.8}} = -1.2 \times 2.8$$

$$r = -3.36$$

Example 3 – Applying Equations of Two Step Problems

Kesler has a cell phone plan that charges him \$0.05 per minute to make phone calls. He also pays a monthly fee of \$4.95. In one month, Kesler 's total phone bill was \$18.75. How many minutes of calls did Kesler make that month?

To solve this problem, create an equation to represent the situation.

Choose a variable to represent the unknown number of minutes, m.

The cost per minutes is \$0.05, so the total cost of phone calls can be represented as $0.05m$.

Then add on the monthly fee of \$4.95, to get the equation, $0.05m + 4.95 = 18.75$

Now solve.

$$\begin{aligned} 0.05m + 4.05 &= 18.75 \\ 0.05m + 4.95 - 4.95 &= 18.75 - 4.95 \\ 0.05m &= 13.80 \\ \cancel{0.05}m \div \cancel{0.05} &= 13.80 \div 0.05 \\ m &= 276 \end{aligned}$$

Kesler used 276 minutes that month.

Practice Questions

1. Solve. Remember to remove all fractions by multiplying by the LCD.

a. $4y - \frac{2}{5} = \frac{3}{5}$

$$(5)4y - (5)\frac{2}{5} = (5)\frac{3}{5}$$
$$20y - 2 = 3$$
$$20y = 5$$
$$y = \frac{5}{20} \text{ or } \frac{1}{4} \text{ or } 0.25$$

b. $1\frac{1}{2} = 4h + \frac{2}{3}$

$$6\left(1\frac{1}{2}\right) = (6)4h + (6)\frac{2}{3}$$
$$9 = 24h + 4$$
$$5 = 24h$$
$$h = \frac{5}{24}$$

c. $\frac{3}{4} - \frac{d}{3} = \frac{3}{8}$

$$(24)\frac{3}{4} - (24)\frac{d}{3} = 24\frac{3}{8}$$
$$18 - 8d = 9$$
$$-8d = -9$$
$$d = \frac{9}{8}$$
$$d = 1\frac{1}{8}$$

d. $0.38 = 6.2 - \frac{r}{1.2}$

$$(1.2)0.38 = (1.2)6.2 - (1.2)\frac{r}{1.2}$$
$$0.456 = 7.44 - r$$
$$-6.984 = -r$$
$$r = 6.984$$

2. The cost of a plain Super Cheese Burger at Sonic is $8.50. For each additional topping there is a charge of $1.35. How many extra toppings are on a burger that costs $13.90? To solve this problem, create an equation to represent the situation. Then, verify the solution.

$$\$8.50 + \$1.35\,t = \$13.90$$
$$\$1.35t = \$5.40$$
$$t = 4$$

There are four extra toppings.

VERIFICATION

$$\$8.50 + \$1.35\,t = \$13.90$$
$$\$8.50 + \$1.35\,(4) = \$13.90$$
$$\$13.90 = \$13.90$$

3. Enzo gets a weekly allowance. On Sunday, Enzo had one quarter of his allowance left. From his leftover allowance, he spent an additional $6.50 playing games at the arcade. He then had $2.25 left. How much is his weekly allowance? To solve this problem, create an equation to represent the situation.

$$\frac{1}{4}x - \$6.50 = \$2.25$$
$$\frac{1}{4}x = \$8.75$$
$$x = \$35$$

Enzo's weekly allowance is $35.00.

Part 3: Solving Equations *a*(*x*+*b*) = *c*

There are a variety of methods to solve equations in the format $a(x + b) = c$. Once a variable has been solved for, check the answer by substitution.

Example 1 – Solving Equations Using The Distributive Property

To isolate the variable in equations of the form $a(x + b) = c$, consider the following steps.

$$\begin{aligned} 4(f-0.4) &= -3.2 \\ 4f-1.6 &= -3.2 \\ 4f-1.6+1.6 &= -3.2+1.6 \\ 4f &= -1.6 \\ f &= -0.4 \end{aligned}$$

Check for $f = -0.4$

$4(-0.4 - 0.4) = -3.2$
$-1.6 - 1.6 = -3.2$
$-3.2 = -3.2$

The solution is correct.

It is possible to solve the same equation by dividing by the constant first.

$$\begin{aligned} 4(f-0.4) &= -3.2 \\ \frac{4(f-0.4)}{4} &= \frac{-3.2}{4} \\ f-0.4 &= -0.8 \\ f-0.4+0.4 &= -0.8+0.4 \\ f &= 0.4 \end{aligned}$$

Practice Questions

1. Solve. Round to one decimal place. Verify the solution for question d.

a. $2(x + 1.5) = 7.6$

$$2x + 3 = 7.6$$
$$2x = 4.6$$
$$x = 2.3$$

b. $3(u - 12.5) = -3.41$

$$3u - 37.5 = -3.41$$
$$3u = 34.09$$
$$= 11.36333$$
$$= 11.4$$

c. $\frac{1}{2}(n-12) = -\frac{1}{4}$

$$\frac{1}{2}n - 6 = -\frac{1}{4}$$
$$4\left(\frac{1}{2}n\right) - (4)6 = 4\left(-\frac{1}{4}\right)$$
$$2n - 24 = -1$$
$$2n = -23$$
$$n = -\frac{23}{2}$$
$$n = -11\frac{1}{2}$$

d. $-3.57 = 3(a + 4.51)$

$$-3.57 = 3a + 13.53$$
$$-17.1 = 3a$$
$$a = -5.7$$

VERIFICATION

$$-3.57 = 3(-5.7 + 4.51)$$
$$-3.57 = -3\ (-1.19)$$
$$-3.57 = -3.57$$

2. Mrs. Waterton used a coupon for eggs at the store. The coupon reduced the cost of each carton of eggs by $0.75. If she paid $6.72 for 3 cartons of eggs, what was the regular price of each carton? To solve this problem, create an equation to represent the situation.

$$3n = \$6.72$$
$$n = \$2.24 \text{ (sale price)}$$
$$\$2.24 + \$0.75 = \$2.99$$

The original price of each carton of eggs was $2.99.

Part 4: Solving Equations *ax+b = cx + d, a(bx + c) = d(ex + f)*

The equations shown in this format contain variables on both sides of the equations. To solve for the variable, it must be isolated.

Example 1 – Solving Equations in the Form *ax = b + cx*

$$0.10d = 0.25(d - 30)$$
$$0.10d = 0.25d - 7.5 \quad \text{Use the distributive property.}$$
$$0.10d - 0.25d = 0.25d - 0.25d - 7.5 \quad \text{Move } d \text{ to the left side.}$$
$$-0.15d = -7.5$$
$$\frac{-0.15d}{-0.15} = \frac{-7.5}{-0.15}$$
$$d = 50$$

Example 2 – Solving Equations in the Form *ax + b = cx + d*

$$35.50 + 4.25d = 24.25 + 5.50d$$
$$35.50 + 4.25d - 4.25d = 24.25 + 5.50d - 4.25d$$
$$35.50 = 24.25 + 1.25d$$
$$35.50 - 24.25 = 24.25 - 24.25 + 1.25d$$
$$11.25 = 1.25d$$
$$\frac{11.25}{1.25} = \frac{1.25d}{1.25}$$
$$9 = d$$

Example 3 – Solving Equations in the Form *a(x + b) = c(x + d)*

$$3(2y - 1) = 4(3y - 1)$$
$$6y - 3 = 12y - 4$$
$$6y - 6y - 3 = 12y - 6y - 4 \quad \text{Subtract } 6y \text{ from both sides.}$$
$$-3 = 6y - 4 \quad \text{Add four to both sides.}$$
$$1 = 6y \quad \text{Divide both sides by 6.}$$
$$\frac{1}{6} = y$$

Practice Questions

1. Solve.

a. $\frac{1}{5}n+\frac{3}{2}=\frac{3}{10}n$

$$10\left(\frac{1}{5}n\right)+10\left(\frac{3}{2}\right)=10\left(\frac{3}{10}n\right)$$
$$2n+15=3n$$
$$15=n$$
$$n=15$$

b. $\frac{3}{4}(d+2)=\frac{2}{3}d$

$$\frac{3}{4}d+\frac{6}{4}=\frac{2}{3}d$$
$$(12)\frac{3}{4}d+(12)\frac{6}{4}=(12)\frac{2}{3}d$$
$$9d+18=8d$$
$$18=-d$$
$$d=-18$$

c. $0.2w - 1.1 = 0.3w$

$$-1.1=0.1w$$
$$w=-11$$

d. $6.2(2g - 3) = 4.2(2g + 3)$

$$12.4g-18.6=8.4g+12.6$$
$$4g=31.2$$
$$g=7.8$$

Lesson 7 Assignment

1. Solve.

a. $4d = -\frac{2}{5}$

$$20d = -2$$
$$d = -\frac{2}{20}$$
$$= -\frac{1}{10}$$

b. $\frac{3.5}{h} = -0.2$

$$3.5 = -0.2h$$
$$h = -17.5$$

c. $\frac{t}{1.6} + 5.9 = -3.2$

$$\frac{t}{1.6} = -9.1$$
$$t = -14.56$$

d. $\frac{2}{5} = \frac{2}{3} - \frac{r}{5}$

$$(15)\frac{2}{5} = (15)\frac{2}{3} - (15)\frac{r}{5}$$
$$6 = 10 - 3r$$
$$-4 = -3r$$
$$r = \frac{4}{3}$$
$$= 1\frac{1}{3}$$

e. $-\frac{4}{5}(q+1) = 1\frac{1}{2}$

$$-\frac{4}{5}q - \frac{4}{5} = \frac{3}{2}$$
$$(10)-\frac{4}{5}q - (10)\frac{4}{5} = (10)\frac{3}{2}$$
$$-8q - 8 = 15$$
$$-8q = 23$$
$$q = \frac{23}{8}$$
$$q = 2\frac{7}{8}$$

f. $4.1 = \frac{1}{9} + \frac{h}{-2}$

$$(18)4.1 = (18)\frac{1}{9} + (18)\frac{h}{-2}$$
$$73.8 = 2 - 9h$$
$$71.8 = -9h$$
$$h = 7.9777 \text{ or } 7.98$$

g. $\frac{1}{5}n = \frac{3}{2} + \frac{3}{10}n$

$$10\left(\frac{1}{5}n\right) + 10\left(\frac{3}{2}\right) = 10\left(\frac{3}{10}n\right)$$
$$2n + 15 = 3n$$
$$15 = n$$
$$n = 15$$

h. $12.4(2h + 3) = 0.9(3h - 2)$

$$24.8h + 37.2 = 2.7h - 1.8$$
$$22.1h = -39$$
$$h = -1.76$$

2. The perimeter of a rectangle is 3.5 times the length. The width is 2.5 cm less than the length. What is the width? To solve this problem, create an equation to represent the situation.

   Perimeter of a rectangle = 2(length) + 2(width)
   Let width = (w)
   Let length = (w + 2.5)
   Perimeter = 2(width + 2.5) + 2(width)
   3.5(w + 2.5) = 2 (w + 2.5) + 2(w)
   3.5w + 8.75 = 2w + 5 + 2w
   3.5w + 8.75 = 4w + 5
   3.75 = 0.5w
   w = 7.5
   The width is 7.5 cm and the length is 10 cm.

3. Mr. Singh paid $14.00 to reserve four tickets to a concert. The total cost of his order including the reservation charge was $153.80. What was the total cost of each ticket? To solve this problem, create an equation to represent the situation.

   t = price per concert ticket.
   $153.80 – $14.00 = 4t
   $139.80 = 4t
   t = $34.95

4. Lateisha has $28.50 in her piggy bank and is saving an additional $8.75 per week by not going to Starbucks. Joan has $104.75 in her piggy bank but is spending $6.50 per week going to Starbucks. In how many weeks will they have the same amount of money? To solve this problem, create an equation to represent the situation.

   $28.50 + $8.75w = $104.75 – $6.50w
   $15.25w = $76.25
   w = 5
   In 5 weeks Lateisha and Joan will have the same amount of money.


LEARNING
ANYTIME
ANYWHERE