Important Concepts . . .

Preview Review

Mathematics Grade 9 TEACHER KEY

W3 - Lesson 11: Properties of Circles

Important Concepts of Grade 9 Mathematics

W1 - Lesson 1 ..... Powers
W1 - Lesson 2 ..... Exponents
W1 - Lesson 3 ..... Rational Numbers
W1 - Lesson 4 ..... Order of Operations
W1 - Lesson 5 ..... Square Roots of Rational Numbers
W1 - Review
W1 - Quiz

W2 - Lesson 6 ..... Graphing Linear Relations
W2 - Lesson 7 ..... Solving Linear Relations
W2 - Lesson 8 ..... Linear Inequalities
W2 - Lesson 9 ..... Polynomials
W2 - Lesson 10 ..... Surface Area of 3D Objects
W2 - Review
W2 - Quiz

W3 - Lesson 11 ..... Properties of Circles
W3 - Lesson 12 ..... Polygons and Scale Diagrams
W3 - Lesson 13 ..... Rotational Symmetry
W3 - Lesson 14 ..... Representing Data
W3 - Lesson 15 ..... Probability
W3 - Review
W3 - Quiz

Materials Required

Paper
Pencil
Calculator

No Textbook Required

This is a stand-alone course.

Mathematics Grade 9
Version 6
Preview/Review W3 - Lesson 11
ISBN: 978-1-927090-00-8

Publisher: Alberta Distance Learning Centre
Written by: Lenee Fyfe
Reviewed by: Danielle Winter

Project Coordinator: Danielle Winter
Preview/Review Publishing Coordinating Team: Julie Reschke

Alberta Distance Learning Centre has an Internet site that you may find useful. The address is as follows: **http://www.adlc.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

ALL RIGHTS RESERVED

Copyright © 2011, by Alberta Distance Learning Centre, 4601-63 Avenue, Barrhead, Alberta, Canada, T7N 1P4. Additional copies may be obtained from Alberta Distance Learning Centre.

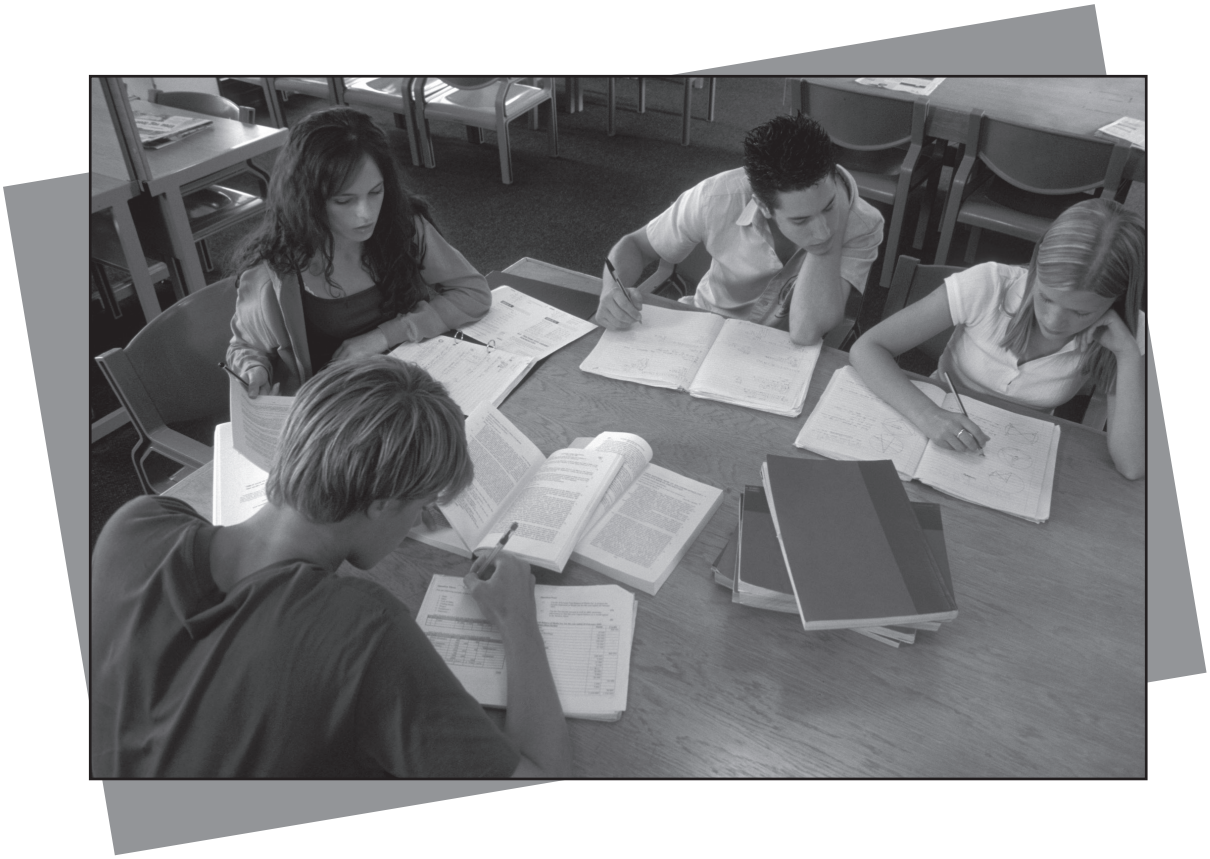
No part of this courseware may be reproduced or transmitted in any form, electronic or mechanical, including photocopying (unless otherwise indicated), recording, or any information storage and retrieval system, without the written permission of Alberta Distance Learning Centre.

Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright law. If cases are identified where this effort has been unsuccessful, please notify Alberta Distance Learning Centre so that appropriate corrective action can be taken.

IT IS STRICTLY PROHIBITED TO COPY ANY PART OF THESE MATERIALS UNDER THE TERMS OF A LICENCE FROM A COLLECTIVE OR A LICENSING BODY.

Preview/Review Concepts for Grade Nine Mathematics

Teacher Key



W3 – Lesson 11:

Properties of Circles

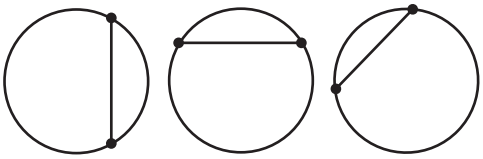
OBJECTIVES

By the end of this lesson, you will be able to:

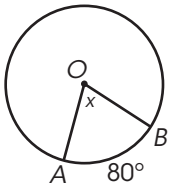
- Solve a given problem involving application of one or more of the circle properties.
- Determine the measure of a given angle inscribed in a semicircle, using the circle properties.
- Explain the relationship among the centre of a circle, a chord and the perpendicular bisector of the chord.

GLOSSARY

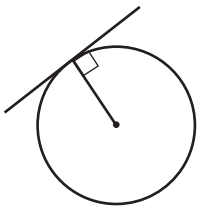
Chord: Is a segment that joins two points of the circle.



Central Angle: A central angle is an angle formed by two intersecting radii such that its vertex is at the center of the circle.



Tangent: A tangent is a line intersecting only one point on the circle; it is perpendicular to the radius.



W2 – Lesson 11: Properties of Circles

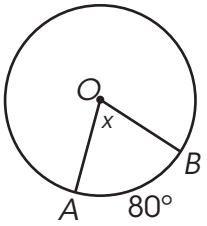
Materials required:

- Paper, Pencil, and Calculator

Part 1: Central Angle Property

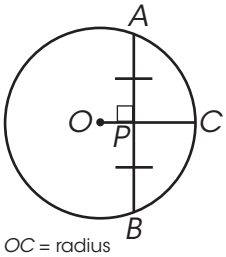
Properties related to angles in circles can be used to solve problems. To solve problems, properties of a circle need to be defined.

The central angle is an angle formed by two radii of a circle.



The central angle is *x*. This angle is formed by the radii lines *AO* and *BO*.

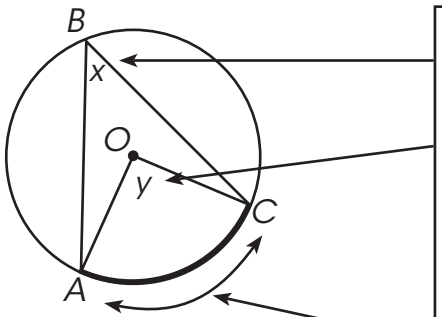
A chord is a line segment with both end points of the line segment falling somewhere on the circle.



OC = radius
AB = chord

The chord *AB* had end points that fall on the circle.

An inscribed angle is an angle formed by two chords that share a common end point, and are subtended by the same arc of the circle.



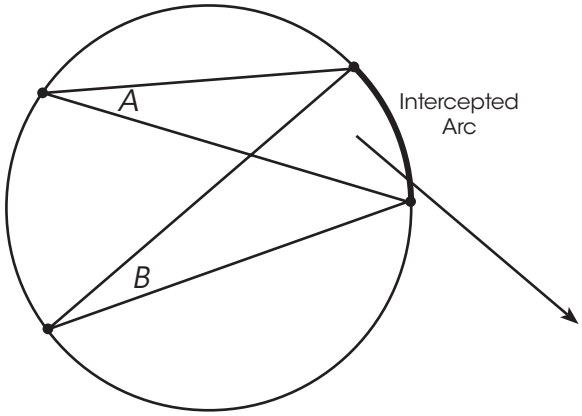
"*x*" is the inscribed angle. It is formed by the two chords *AB* and *BC*.

"*y*" is the central angle.

The central angle is twice the value of the inscribed angle.

Both angles are subtended by the same arc of the circle.

Any inscribed angles on a circle subtended by the same arc are congruent.



Angle *A* = Angle *B*

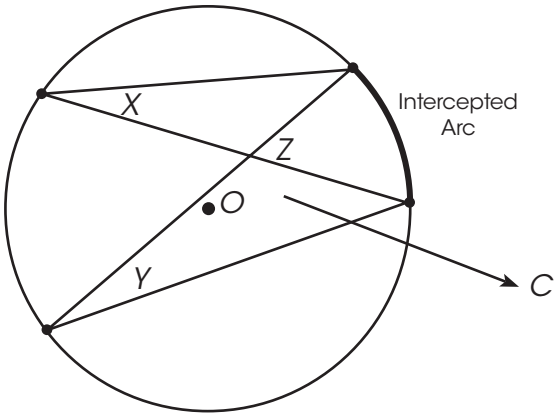
The inscribed angles *A* and *B* are congruent.

Angle *A* and Angle *B* are both 35°.

The measure of the central angle is twice the value of an inscribed angle subtended by the same arc. The central angle would be 70°.

Example 1

"*O*" is the centre of the circle.
$\angle X$ is 35°.



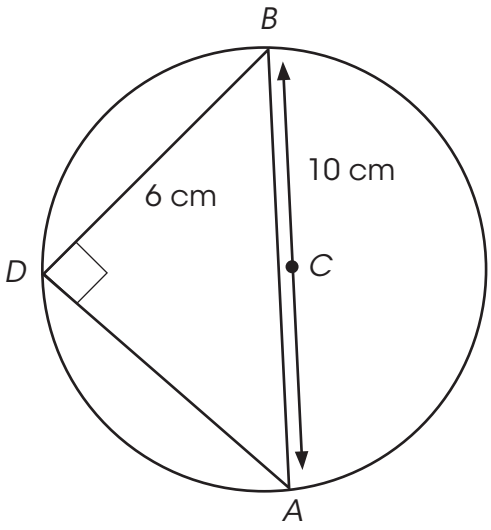
What is the measure of $\angle Y$?
Because $\angle X$ and $\angle Y$ share the same subtend arc, $\angle X = \angle Y$. So $\angle Y = 35°$.

What is the measure of $\angle Z$?

Angle *A* = Angle *B*

The measure of the central angle is equal to twice the measure of the inscribe angle. So the measure of $\angle Z$ is 70°.

Example 2



Point *C* is the center of the circle. The diameter of this circle is 10 cm. The diameter is represented by the line *AB*. There are two chords in this circle.

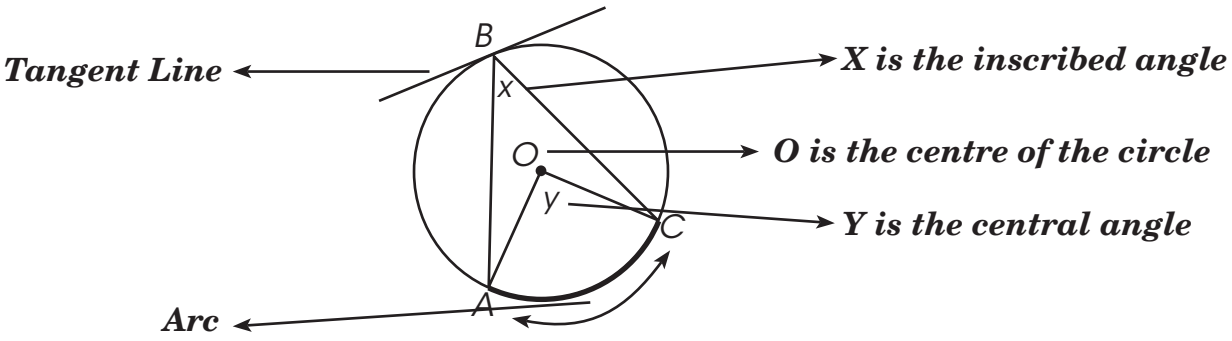
a. What is the length is the chord *AD*?

Because triangle *ADB* is a right angled triangle, to find the length of chord *AD*, use the Pythagorean Theorem ($a^2 + b^2 = c^2$) to find the length of *AD*.

$$AD^2 + BD^2 = AB^2$$
$$AD^2 + 6^2 = 10^2$$
$$AD^2 + 36 = 100$$
$$AD^2 = 100 - 36$$
$$AD^2 = 64$$
$$AD^2 = \sqrt{64}$$
$$AD^2 = 8 \text{ cm}$$

Practice Questions

1. Label the following on the diagram below: *centre of the circle*, *tangent line*, *arc*, *inscribed angle*, and *central angle*.



2. Determine the following:

   a. The inscribed angle on a given circle is 45°. What is the measurement of the central angle?

   90°

   b. The inscribed angle on a given circle is 25°. What is the measurement of the central angle?

   50°

   c. The central angle on a given circle is 144°. What is the measurement of the inscribed angle?

   72°

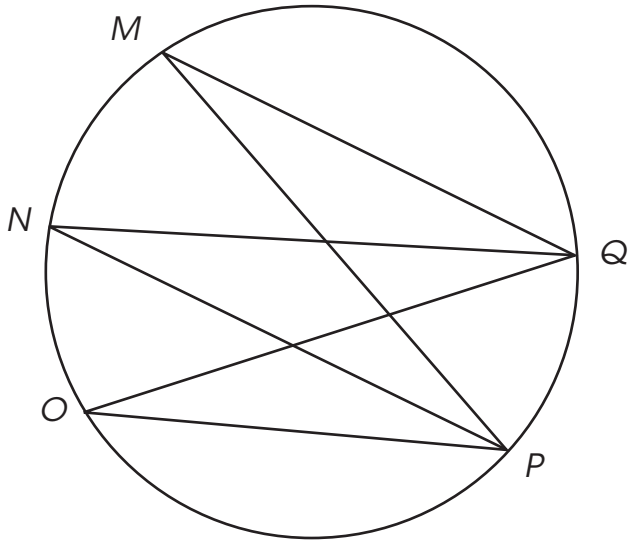
   d. The central angle on a given circle is 120°. What is the measurement of the inscribed angle?

   60°

   e. Two inscribed angles share the same arc, therefore they are congruent. If the value of one angle is 73°, what is the value of the other inscribed angle?

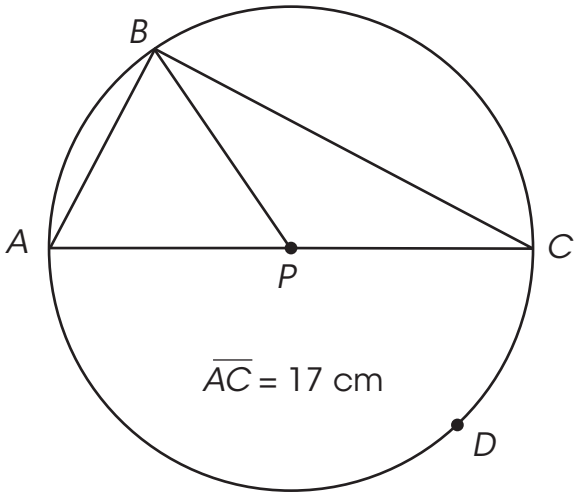
   73°

3. Using inscribed angle properties, determine which 3 angles are congruent. Circle the three congruent angles below.



(∠M) (∠N) (∠O) ∠P ∠Q

4. Point P is the center of the circle. The diameter of this circle is 17 cm. The diameter is represented by the line AC. There are two chords in this circle. Chord BC is 15 cm long.



a. What is the measure of $\angle ABC$?

90°

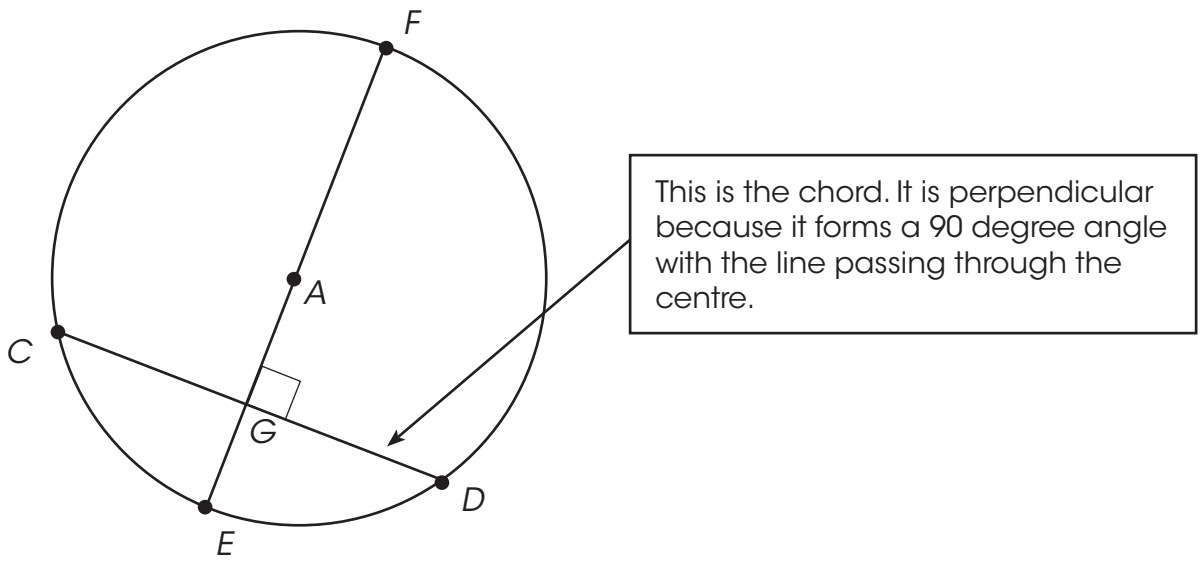
b. What is the length of the chord AB?

$a^2 + b^2 = c^2$
$a^2 + 15^2 = 17^2$
$a^2 = 64$
$a = 8cm$
Chord AB is 8 cm long.

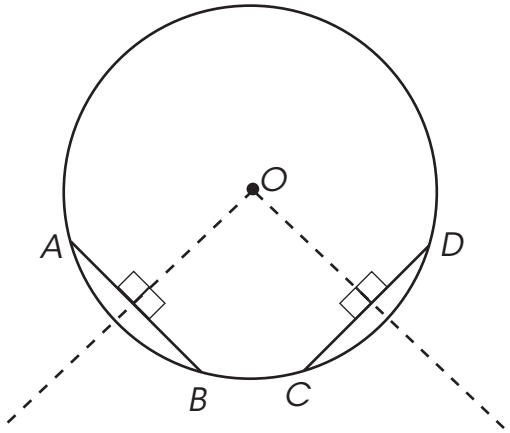
Part 2: Chord Properties

If a line passes through the centre of a circle and intersects a chord at right angles, then the line bisects the chord.

The perpendicular bisector of a chord passes through the center of the circle.

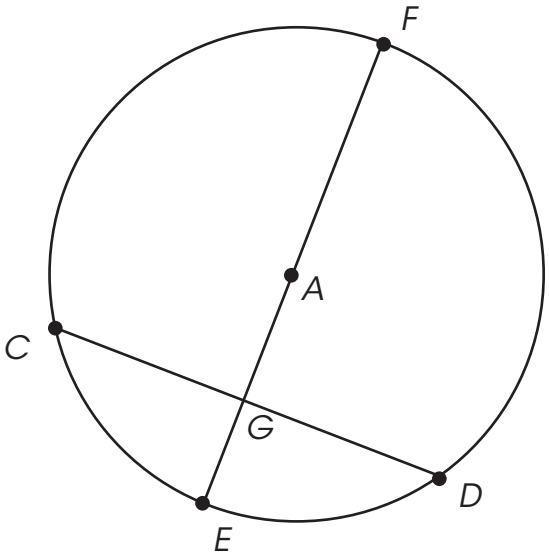


When a bisector of a chord passes through the centre of the circle, the bisector is perpendicular to the chord.



Example 1

The radius AE bisects chord CD. AG measures 4 mm. Chord CD measures 14 mm. What is the radius of the circle?



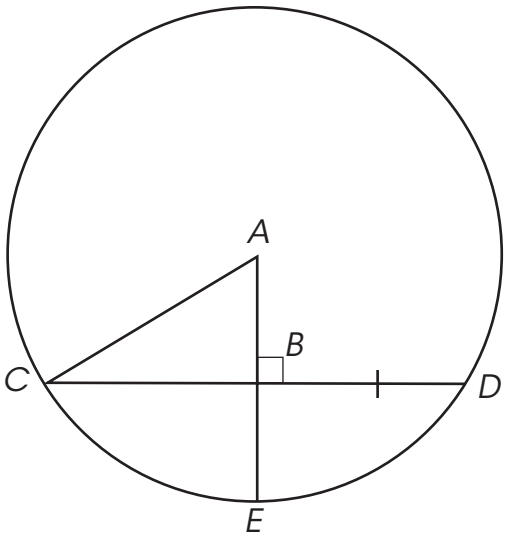
Drawing a radius from AD. This will form a right angled triangle. Now apply Pythagorean Theorem to solve.

$$a^2 + b^2 = c^2$$
$$4^2 + 7^2 = AD^2$$
$$16 + 49 = AD^2$$
$$65 = AD^2$$
$$\sqrt{65} = AD$$
$$8.1 = AD$$

The radius of the circle is 8.1 mm.

Example 2

AE bisects chord CD. The diameter of the circle is 30 cm long. Chord CD measures 24 cm. What is the length of AB?



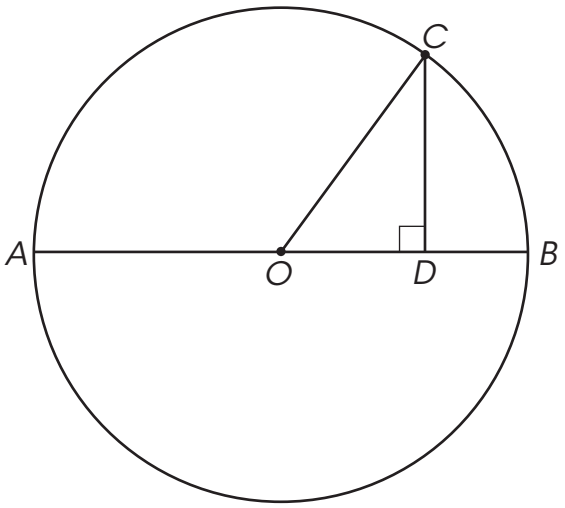
Since triangle ABC is a right angled triangle, it is possible to solve for the length of AB using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$12^2 + AB^2 = 15^2$$
$$144 + AB^2 = 225$$
$$AB^2 = 81$$
$$AB = \sqrt{81}$$
$$AB = 9 \text{ cm}$$

$\overline{AB}$ is 9 cm long.

Practice Questions

1. The center of the circle is O.



$AO = 5$ cm
$DB = 2$ cm

a. What is the length of OC?

OC = AO
= 5 cm

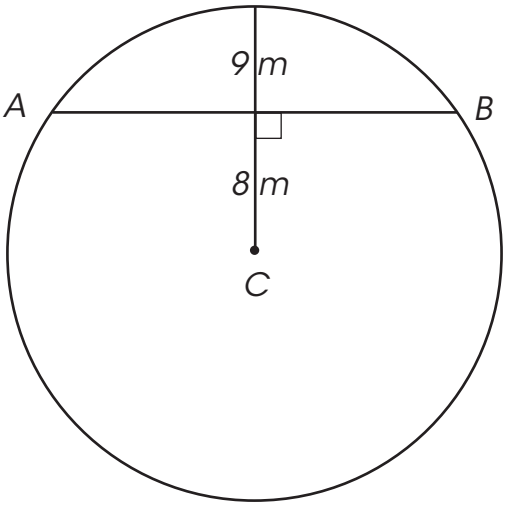
b. What is the length of OD?

OD = OB – DB = 5 – 2 = 3 cm

c. What is the length of CD?

$$a^2 + b^2 = c^2$$
$$3^2 + b^2 = 5^2$$
$$9 + b^2 = 25$$
$$b^2 = 16$$
$$b = 4 \text{ cm}$$

2. Look at the following diagram. The chord is 80 m long. The radius is 17 m long



a. What is the distance between points C and A?

$$a^2 + b^2 = c^2$$
$$8^2 + 40^2 = c^2$$
$$64 + 1600 = c^2$$
$$1664 = c^2$$
$$c = 40.8 \text{ m}$$

The distance between C and A is 40.8 m

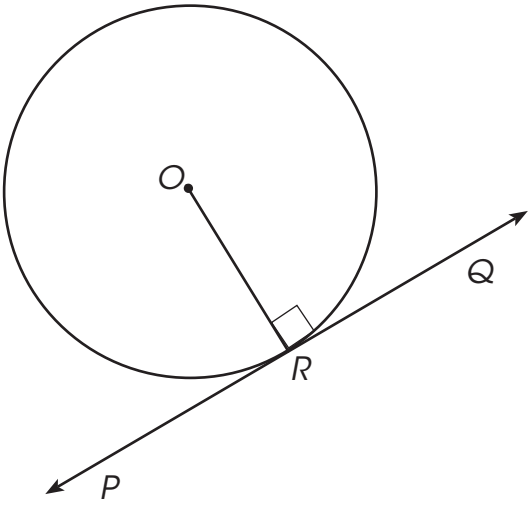
b. What is the distance between points C and B?

Same as above. The distance between C and A is 40.8 m

Part 3: Tangent Properties

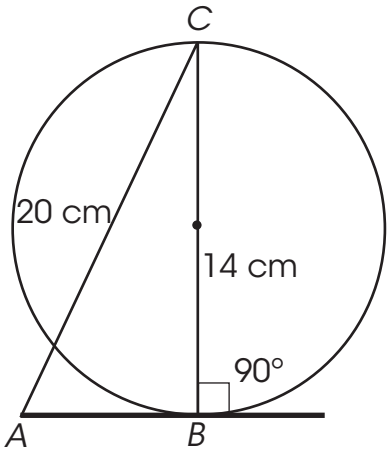
A tangent is a line that touches a circle at exactly one point. The point where the line touches the circle is called the point of tangency.

The properties of tangents to a circle can be used to solve problems.



Tangent to a Circle: A tangent to a circle is perpendicular to the radius at the point of tangency.

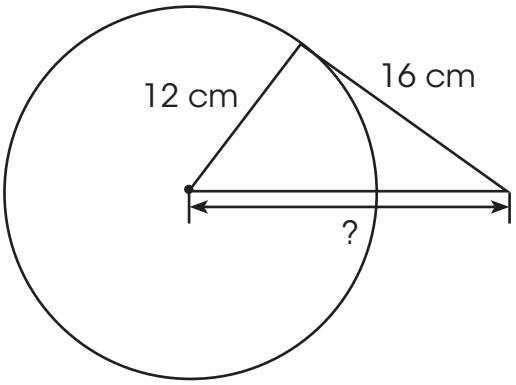
R is the point of tangency.



Tangent Chord Relationship: A chord that is drawn perpendicular to a tangent of a circle, at the point of tangency, will contain the centre of the circle. This is the diameter.

Example 1

What is the measurement of the unknown length?

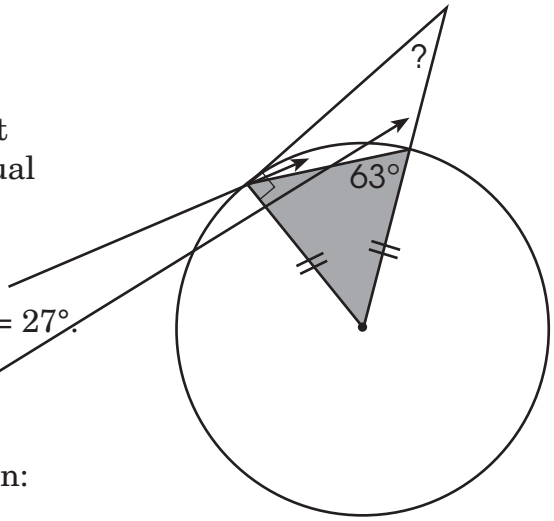


Since a tangent to a circle is perpendicular to the radius at the point of tangency, this creates a right triangle. Use the Pythagorean Theorem to find the unknown side.

$$a^2 + b^2 = c^2$$
$$12^2 + 16^2 = c^2$$
$$144 + 256 = c^2$$
$$400 = c^2$$
$$\sqrt{400} = c$$
$$c = 20$$

The unknown side is 20 cm.

Example 2



The smaller triangle is an isosceles triangle because it contains two radii of the circle and the radii are of equal length. Therefore, the other angle is also 63°.

Drawing a tangent line creates a 90° angle with the centre of the circle. If one angle is 63°, then 90° – 63° = 27°.

A straight line is 180°, therefore 180° – 63° = 117°

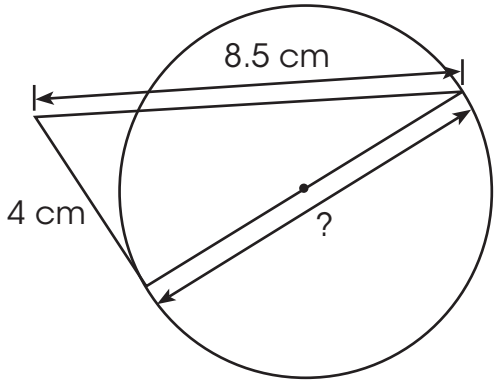
Since the sum of the angles in any triangle is 180, then:
27° + 117° + unknown angle = 180
180° – 117° – 27° = 36°

The unknown angle is 36°.

Practice Questions

1. Find the missing length.

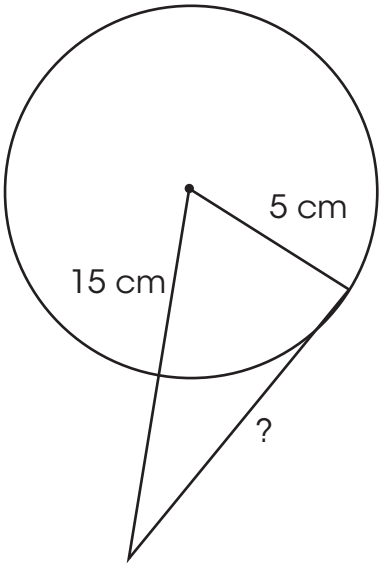
a.



$a^2 + b^2 = c^2$
$4^2 + b^2 = 8.5^2$
$16 + b^2 = 72.25$
$b^2 = 56.25$
$b = 7.5$

The missing length is 7.5 cm.

b.

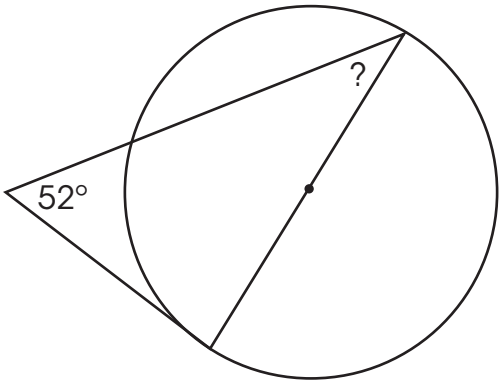


$a^2 + b^2 = c^2$
$5^2 + b^2 = 15^2$
$25 + b^2 = 225$
$b^2 = 200$
$b = 14.14$

The missing length is 14.14 cm.

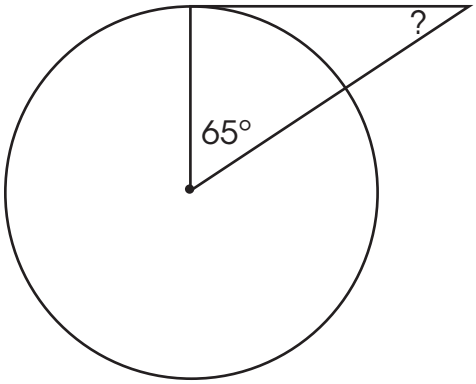
2. Find the missing length.

a.



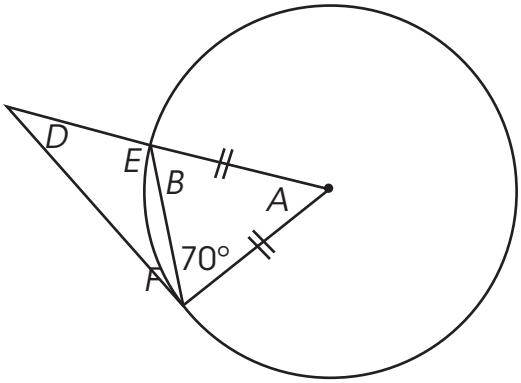
180° – 52° – 90° = 38°

b.



180° – 65° – 90° = 25°

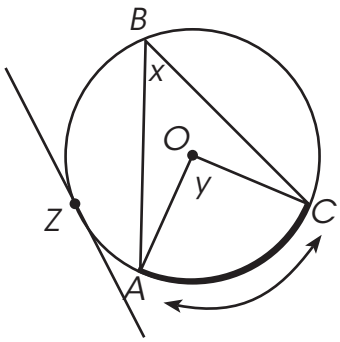
c.



∠A 40° because 180° – 70° – 70° = 40°
∠B 70° because it is the same as the congruent angle.
∠D 50° because 180° – 70° – 70° = 40°
∠E 110° because 180° – 70° = 110°
∠F 20° because 90° – 70° = 20°

Lesson 11 Assignment

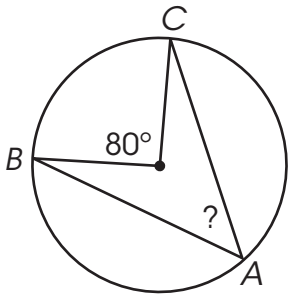
1. Label the following parts of the circle:



X refers to the ***inscribed angle***
Y refers to the ***central angle***
O refers to the ***centre of the circle***
Z refers to the ***point of tangency***
The arrow refers to the ***arc***

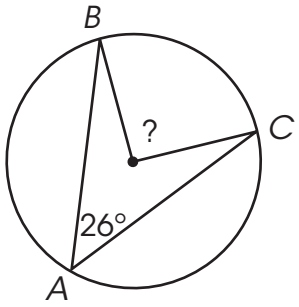
2. Find the measure of the angle indicated.

a.



The inscribed angle is half the central angle; the missing angle is 40°.

b.



The central angle is double the inscribed angle; the missing angle is 52°.

3. Answer the following:

   a. The inscribed angle on a given circle is 33°. What is the measurement of the central angle?

   66°

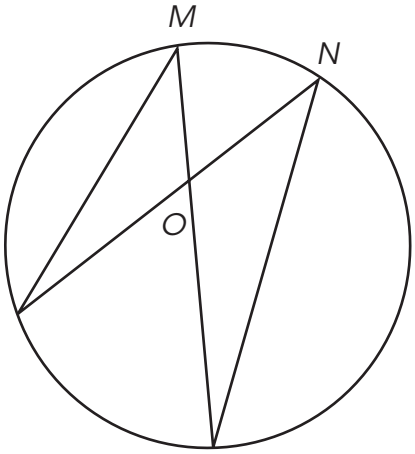
   b. The central angle on a given circle is 128°. What is the measurement of the inscribed angle?

   64°

   c. Two inscribed angles share the same arc, therefore they are congruent. If the value of one angle is 61°, what is the value of the other inscribed angle?

   61°

4. If the value of *O* is 96°, what is the value of *M*? What is the value of *N*?



The central angle is double the inscribed angle. Therefore, both M and N are 48°.

5. The center of the circle is *O*.
   AO = 6 cm
   DB = 3 cm

   a. What is the length of *OC*?

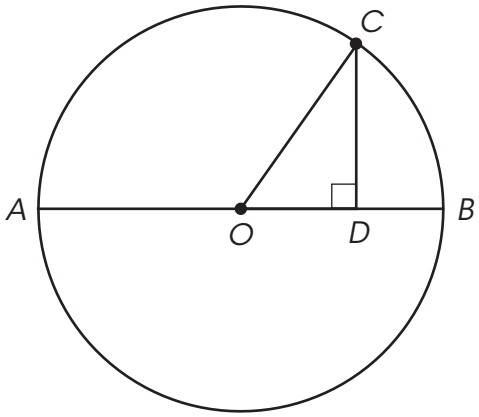
   AO = CO = 6 cm

   b. What is the length of *OD*?

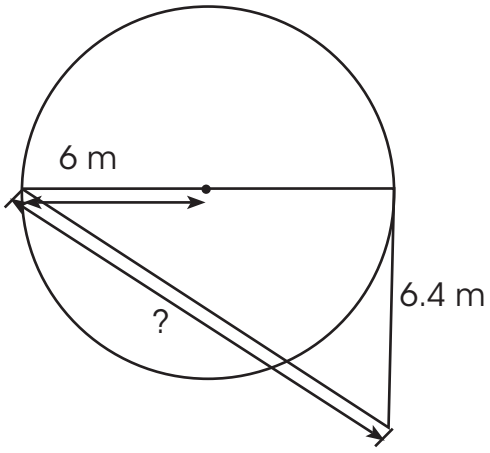
   6 – 3 = 3cm

   c. What is the length of *CD*?

   $3^2 + a^2 = 6^2$, a = 5.20 cm



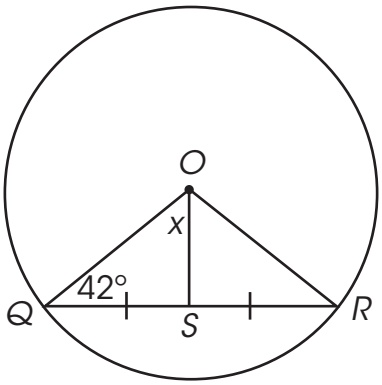
6. Find the missing length.



$$a^2 + b^2 = c^2$$
$$12^2 + 6.4^2 = c^2$$
$$144 + 40.96 = c^2$$
$$184.96 = c^2$$
$$c = 13.6$$

The missing length is 13.6 m.

7. Line segment OS bisects the chord QR. Q is 42°. O is the centre of the circle. What is the value of R? What is the value of x?



Since QOR is an isosceles triangle, OQS is congruent to ORS. Therefore, the value of R is 42°.

Every triangle has 180°.
180° – 42° – 90° = 48°
The value of x is 48°.

LEARNING
ANYTIME ANYWHERE