Important Concepts . . .

Preview Review

Mathematics Grade 9 TEACHER KEY

W1 - Lesson 2: Exponents

Important Concepts of Grade 9 Mathematics

Materials Required

Paper
Pencil
Calculator

No Textbook Required

This is a stand-alone course.

Mathematics Grade 9
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Preview/Review W1 - Lesson 2
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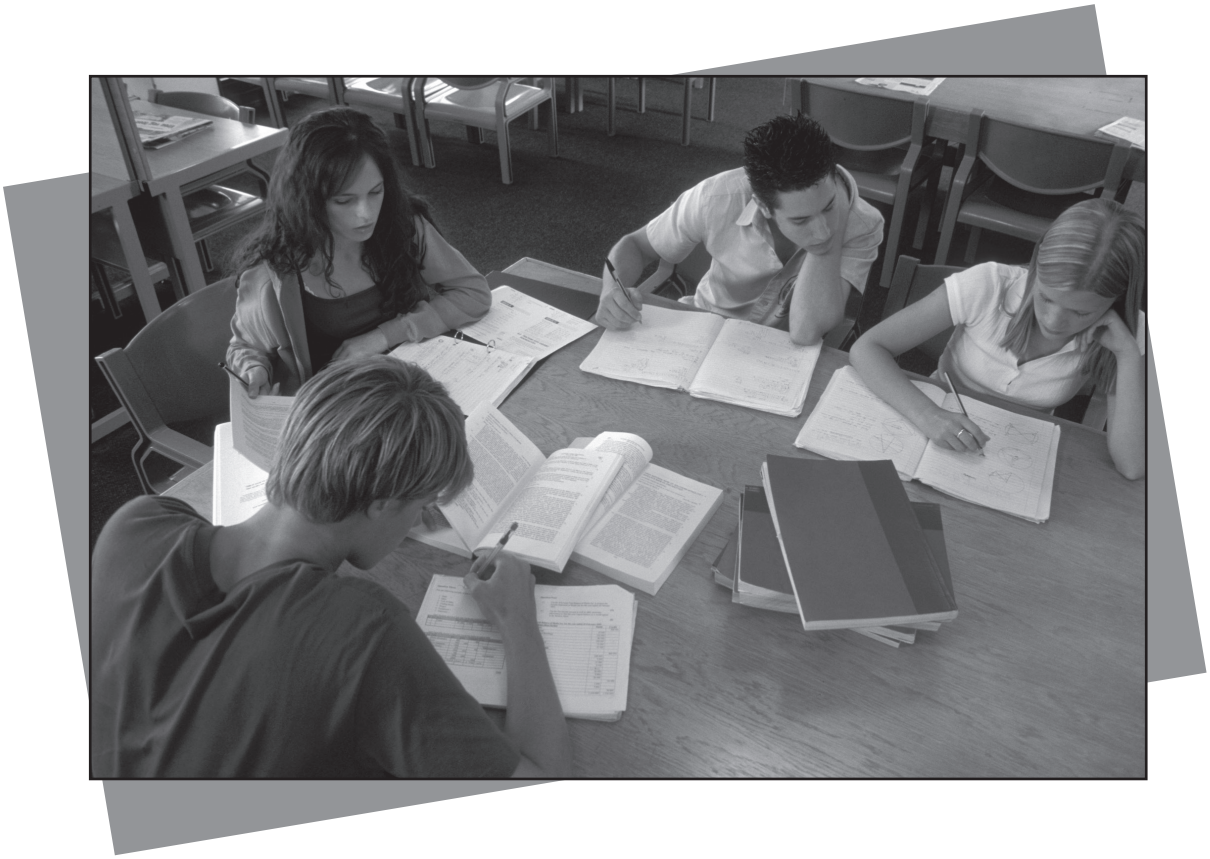
Preview/Review Concepts for Grade Nine Mathematics

Teacher Key



W1 – Lesson 2:

Laws of Exponents

OBJECTIVES

By the end of this lesson, you will be able to:

- Explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents.
- Evaluate a given expression by applying the exponent laws.

GLOSSARY

Product Law: $x^a x^b = x^{a+b}$

Power of a Product Law: $\left(x^a\right)^b = x^{ab}$

Division law: $\frac{x^a}{x^b} = x^a x^{-b} = x^{a-b}$

Zero Exponent Law: $x^0 = 1$ (if $x \neq 0$)

W1 – Lesson 2: Laws of Exponents

Materials required:

- Paper, Pencil, and Calculator

Part 1: Multiplication of Powers – The Product Law

In the expression $(x^5)(x^6)$, notice that the bases are the same. This expression can be simplified into one exponent instead of two: the key is to count factors.

So,

$$x^5 = (x)(x)(x)(x)(x) \text{ and } x^6 = (x)(x)(x)(x)(x)(x)$$

Multiply them together:

$$(x^5)(x^6) = (x)(x)(x)(x)(x)(x)(x)(x)(x)(x)(x) = x^{11}$$

There are five x factors from x^5, and six x factors from x^6, which yields 11 x factors in total.

The **Product Law** can then be written as:

$$x^a x^b = x^{a+b}$$

Example 1

Simplify $(5^4)\ (5^3)$

$= (5)(5)(5)(5)\ \ (5)(5)(5)$

$= (5)(5)(5)(5)(5)(5)(5)$

$= 5^7$

Note: The rule above does not work when multiplying powers with a different base. For instance,

$$(x^3)(y^4) = (x)(x)(x)(y)(y)(y)(y)$$

When writing out the powers, there is no way to combine them.

However, if the bases are different but the exponents are the same, then they can be combined.

Example:

$$(x^3)(y^3) = (x)(x)(x)(y)(y)(y) = (xy)^3$$

So combining powers with different bases can be written as:

$$x^a x^a = (xy)^a$$

Example 1

Simplify $(4)^3 \quad (5)^3$

$= (4)(4)(4) \quad (5)(5)(5)$

$= (4 \times 5)^3$

$= 20^3$

Practice Questions

1. Simplify each of the following.

   a. $(x^3)(x^6)$

   x^9

   b. $(3^5)(3^2)$

   3^7

   c. $-(8^4)(8^3)$

   -8^7

   d. $(9^6)(9^5)$

   9^{11}

2. Simplify each of the following.

   a. $(d^3)(f^5)$

   $(df)^5$

   b. $(8^4)(6^4)$

   48^4

   c. $(5^6)(3^6)$

   15^6

   d. $(6^4)(7^4)$

   42^4

Part 2: Division of Powers

When dividing exponents, subtract the exponents when the bases are the same.

To simplify $x^8 \div x^6$, subtract the exponents.

$x^8 \div x^6 = x^{8-6} = x^2$

So the **Division Law** can be written as: $\frac{x^a}{x^b} = x^a x^{-b} = x^{a-b}$

Example 1

Simplify $(6^6) \div (6^3)$

$= (6)(6)(6)(6)(6)(6) \div (6)(6)(6)$

$= (6)^{6-3}$

$= 6^3$

When dividing powers with different bases but a common exponent, this exponent can be applied to both the numerator and denominator. For instance, $x^3 \div y^3$.

$= \frac{(x)(x)(x)}{(y)(y)(y)}$

$= \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right)$ which is $\left(\frac{x}{y}\right)^3$.

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

Example 2

Simplify $(4)^6 \div (5)^6$

$= \frac{4^6}{5^6}$

$= \left(\frac{4}{5}\right)^6$

Practice Questions

1. Simplify each of the following.

   a. $(j^5) \div (j^4)$

   j

   b. $(2^5) \div (2^2)$

   2^3

   c. $-(5^5) \div (5^3)$

   -5^2

   d. $(9^9) \div (9^5)$

   9^4

2. Simplify each of the following.

   a. $(h^5) \div (k^5)$

   $\left(\frac{h}{k}\right)^5$

   b. $(3^4) \div (6^4)$

   $\left(\frac{3}{6}\right)^4$

Part 3: Power of a Power

With an expression line $(x^5)^4$, the rule can be worked out by counting.

$$(x^5)^4 = (x^5)(x^5)(x^5)(x^5)$$

Write this as an array:

$$\begin{array}{l} x^5 = (x)\ (x)\ (x)\ (x)\ (x) \\ x^5 = (x)\ (x)\ (x)\ (x)\ (x) \\ x^5 = (x)\ (x)\ (x)\ (x)\ (x) \\ x^5 = (x)\ (x)\ (x)\ (x)\ (x) \end{array}$$

There are 20 factors of x, therefore, $(x^5)^4 = x^{20}$

So the **Power of a Power Rule** can be written as:

$$\left(x^a\right)^b = x^{ab}$$

Note the difference between the **Power Rule**.

$$x^5 x^4 = (xxxxx)(xxxx) = x^9$$

Example 1

Simplify $(4^3)^5$
$= (4^3)\ (4^3)\ (4^3)\ (4^3)\ (4^3)$ or $4^{3\times5}$
$= 4^{15}$

Example 2

Simplify $(-5^3)^2$
$= (-5^3)\ (-5^3)$ or $(-5)^{3\times2}$
$= (-5)^6$

Lesson 2 Assignment

1. Evaluate each of the following.

   a. $-(5)^6$

   –15 625

   b. 4^6

   4096

2. Simplify each of the following.

   a. $(3^3)(3^4)$

   3^7

   b. $(4^5)(4^6)$

   4^{11}

   c. $(h^7) \div (h^5)$

   h^2

   d. $(6^6) \div (6^4)$

   6^2

3. Evaluate the following powers, then circle the lesser value in each set.

   a. $2(2^2)^3$ or $3(2^3)^2$

   $(2)(2)^6$ or $(3)(2)^6$

   128 or 192

   b. $6^3 \times 6^4$ or $3^6 \times 3^6$

   6^7 or 3^{12}

   279 936 or 531 441

4. Simplify the following.

   a. $(5^4)^5$

   5^{20}

   b. $(4^6)^5$

   4^{30}

5. Simplify the following.

a. $\dfrac{(8^2)(8^5)(8^{-3})}{(8^2)}$

$= \left(\dfrac{8^4}{8^2}\right) = 8^2$

b. $\dfrac{(6^7)(6^2)}{(6^2)}$

$= \left(\dfrac{6^9}{6^2}\right) = 6^7$

c. $\dfrac{\left[(-5)^2\right]^4}{(-5)^3}$

$= \dfrac{(-5)^8}{(-5)^3} = (-5)^5$

d. $\left[(a^2)(a^3)\right]^2$

$= (a^5)^2 = a^{10}$

6. Write the following as a single power and then evaluate the expression.

a. $2^5 \times 2^7$

$= 2^{12}$
$= 4096$

b. $3(mn)^4$ where $m = 6$ and $n = 5$

$= 3m^4n^4 = 3\,(6)^4(5)^4 = (3)(1296)(625) = 2\,430\,000$

c. $8(3^5)^2$

$= (8)(3^{10}) = 472\,392$


LEARNING
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ANYWHERE