Important Concepts . . .

Preview Review

Mathematics Grade 9 TEACHER KEY

W2 - Lesson 10: Surface Area of 3D Objects

Important Concepts of Grade 9 Mathematics

Materials Required

Paper
Pencil
Grid Paper
Calculator
3D Solids

No Textbook Required

This is a stand-alone course.

Mathematics Grade 9
Version 6
Preview/Review W2 - Lesson 10
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Preview/Review Concepts for Grade Nine Mathematics

Teacher Key

W2 – Lesson 10:

Surface Area of 3D Objects

OBJECTIVES

By the end of this lesson, you will be able to:

- Determine the surface area of a given composite 3-D object.
- Solve a given problem involving surface area.

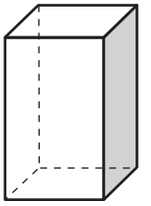
GLOSSARY

Surface Area: Surface area is the total area of all faces, or surfaces, of a three-dimensional figure. It is the total number of square units needed to cover the outside a three-dimensional figure.

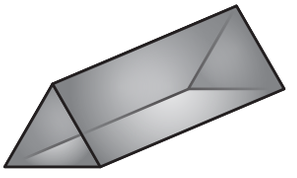
3D Object: A three-dimensional object is a figure that has length, width and depth.

Cylinder: A cylinder is a 3D object having two congruent (identical) circular bases that are parallel.

Rectangular Prism: A solid (3-dimensional) object which has six faces that are rectangles.



Triangular Prism: A solid (3-domensional) with two bases that are congruent (identical), parallel triangles and all other faces are rectangles.



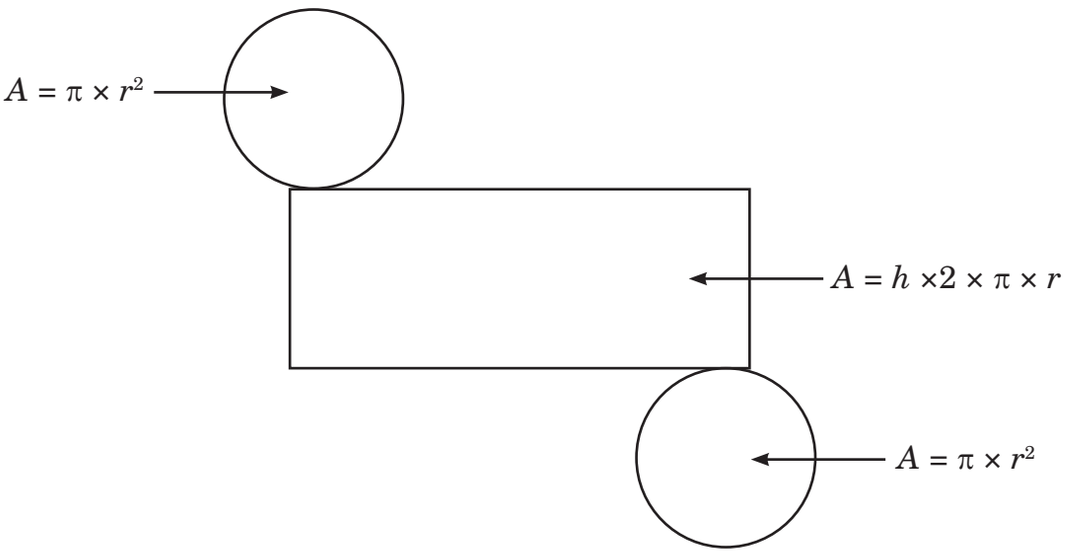
W2 – Lesson 10: Surface Area of 3D Objects

Materials required:

- Paper, pencil, grid paper, calculator, 3D solids of: cylinder, rectangular prism and triangular prism

Part 1: Finding Surface Area of 3D Objects

Finding the Total Surface Area of the Cylinder



So the total surface area of the cylinder or *SA* is:

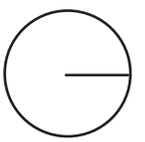
$SA = (\pi \times r^2) + (\pi \times r^2) + (h \times 2 \times \pi \times r)$

Or to shorten it:

$SA = 2\pi r^2 + 2\pi rh$

Recall Facts!

Radius = Diameter ÷ 2

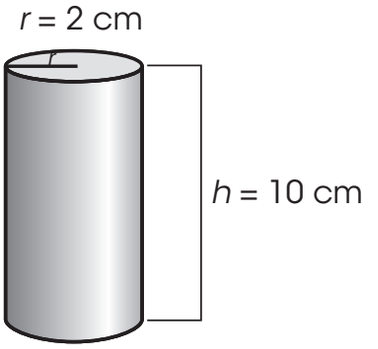


r = radius, the distance from the center of the circle to the outside.

Example 1

Find the surface area of a cylinder with a radius of 2 cm, and a height of 10 cm.

$SA = 2\pi r^2 + 2\pi rh$
$SA = (2 \times 3.14 \times 2^2) + (2 \times 3.14 \times 2 \times 10)$
$SA = (6.28 \times 4) + (6.28 \times 20)$
$SA = (25.12) + (125.6)$
Surface Area = 150.72 cm^2



Example 2

Finding Surface Area of Rectangular Prisms

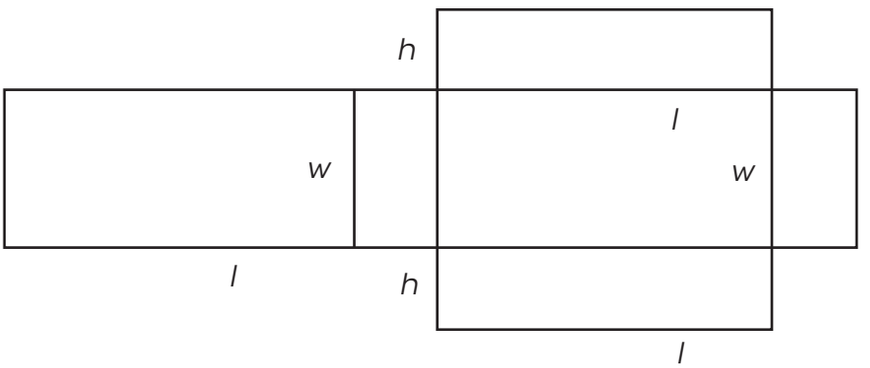
Top Face = Bottom Face
Front Face = Back Face
Right Face = Left Face

So to find the area of each face:

Recall Facts!

Recall Facts:
The area of a rectangle is:
$A = l \times w$

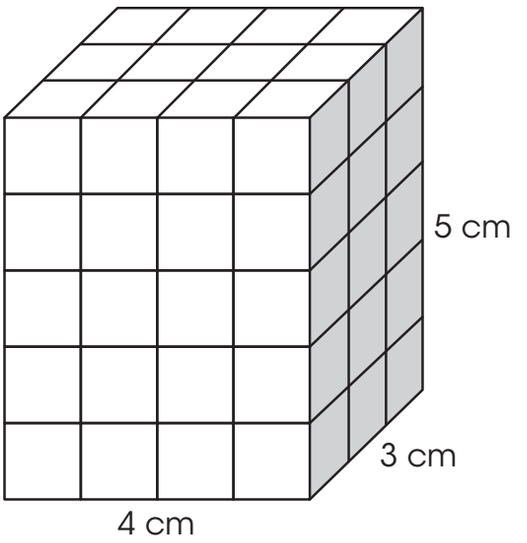
Top = $l \times w$ or (lw)
Bottom = $l \times w$ or (lw)
Front = $h \times$ x l or (hl)
Back = $h \times l$ or (hl)
Left = $h \times w$ or (hw)
Right = $h \times w$ or (hw)



Total Surface Area = $(lw) + (lw) + (hl) + (hl) + (hw) + (hw)$
Total Surface Area = $2(lw) + 2(lh) + 2(wh)$
Total Surface Area = $2(lw + hl + hw)$

Note: Any one of these formulas work for calculating the total surface area of a prism. Choose the one that works best for you.

Find the surface area for this rectangular prism.



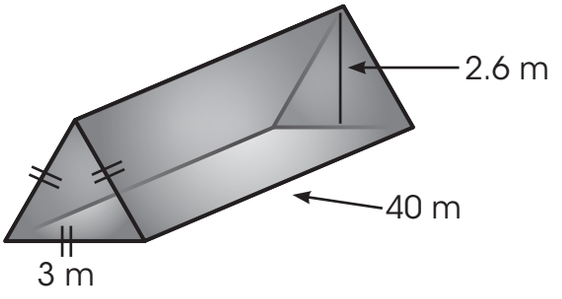
$SA = 2(lw) + 2(lh) + 2(wh)$
$SA = 2(4 \times 3) + 2(4 \times 5) + 2(3 \times 5)$
$SA = 2(12) + 2(20) + 2(15)$
$SA = 24 + 40 + 30$
$SA = 94 \text{ cm}^2$

Example 3

Finding Surface Area of Triangular Prisms With the Height given

The formula is: $SA = 2(\frac{1}{2} \text{ base} \times \text{height}) + 3(\text{length} \times \text{width})$

Find the surface area of a triangular prism if the height of the triangle is 2.6 m, and each side of the triangle measures 8 m. The prism is 40 m long.



$SA = 2\,(\frac{1}{2}\ b \times h) + 3\,(l \times w)$

$SA = 2\,(\frac{1}{2} \times 3 \times 2.6) + 3\,(40 \times 3)$

$SA = 7.8 + 360$

$SA = 367.8 \text{ m}^2$

Finding Surface Area of Triangular Prisms With the Height given

If the height of the triangular base is not given, then you must first calculate the missing height, using The Pythagorean Theorem.

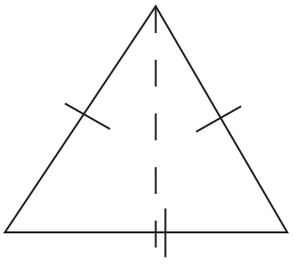
Remember, the Pythagorean Theorem is:

$a^2 + b^2 = c^2$

- This only works on right triangles.
- "C" always refers to the side opposite to the right angle; it is always the hypotenuse.

Find the surface area of a giant Candy Bar. Each side of the triangle measures 4 cm. The prism is 21 cm long. No triangle height has been given, so it must first be calculated, using the Pythagorean Theorem.

Take the equilateral triangle and split it up into 2, right angled triangles, like this:



Note: Each side is 4 cm long, so half of a side will be 2 cm.

$a^2 + b^2 = 10^2$

$2^2 + b^2 = 4^2$

$4 + b^2 = 16$

$b^2 = 12$

$b = 3.46$ cm

Then calculate the surface area of the entire Candy Bar:

$$SA = 2\left(\frac{1}{2}\ b \times h\right) + 3\ (l \times w)$$

$$SA = 2\left(\frac{1}{2} \times 4 \times 3.46\right) + 3\ (21 \times 4)$$

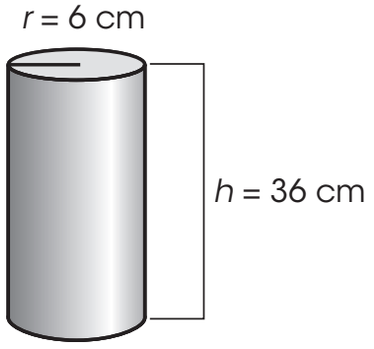
$SA = 13.84 + 252$

$SA = 265.84\ \text{cm}^2$

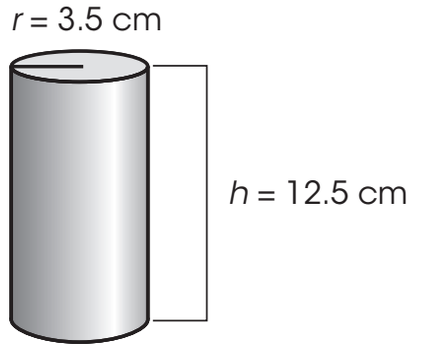
Practice Questions

1. Find the surface area of the following cylinders.

a.



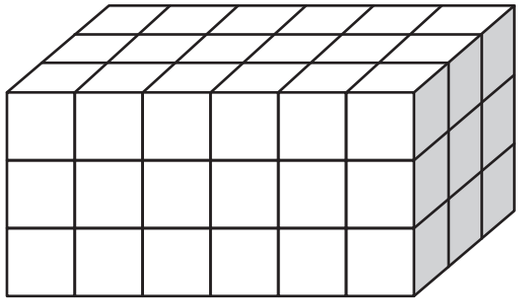
b.



$SA = 2\pi r^2 + 2\pi rh$
$= 2(3.14 \times 6^2) + (2 \times 3.14 \times 6 \times 36)$
$= 226.08 + 1356.48$
$= 1582.56 \text{ cm}^2$

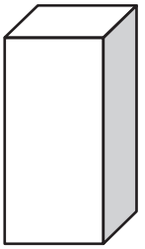
$SA = 2\pi r^2 + 2\pi rh$
$= 2\,(3.14 \times 3.5^2) + (2 \times 3.14 \times 3.5 \times 12.5)$
$= 76.93 + 274.75$
$= 351.68 \text{ cm}^2$

2. Find the surface area for this rectangular prism.



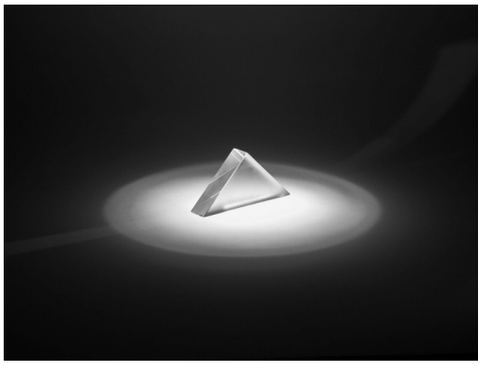
$SA = 2(lw) + 2(lh) + 2(wh)$
$= 2(6 \times 3) + 2(6 \times 3) + 2(3 \times 3)$
$= 36 + 36 + 18$
$= 90 \text{ units}^2$

3. Find the surface area of a rectangular prism with a length of 4 cm, a width of 5 cm, and a height of 10 cm.



$$SA = 2(lw) + 2(lh) + 2(wh)$$
$$= 2(4 \times 5) + 2(4 \times 10) + 2(5 \times 10)$$
$$= 40 + 80 + 100$$
$$= 220 \text{ cm}^2$$

4. Find the surface area of this glass prism if it is 66 mm high and each side of the triangle is 110 mm long.



$$a^2 + b^2 = c^2$$
$$55^2 + b^2 = 110^2$$
$$3025 + b^2 = 12\,100$$
$$b^2 = 9075$$
$$b^2 = 95.26 \text{ mm}$$

$$SA = 2(\frac{1}{2} b \times h) + 3(l \times w)$$
$$SA = 2(\frac{1}{2} \times 110 \times 95.26) + 3(110 \times 66)$$
$$SA = 10\,478.6 + 21\,780$$
$$SA = 32\,258.6 \text{ mm}^2$$

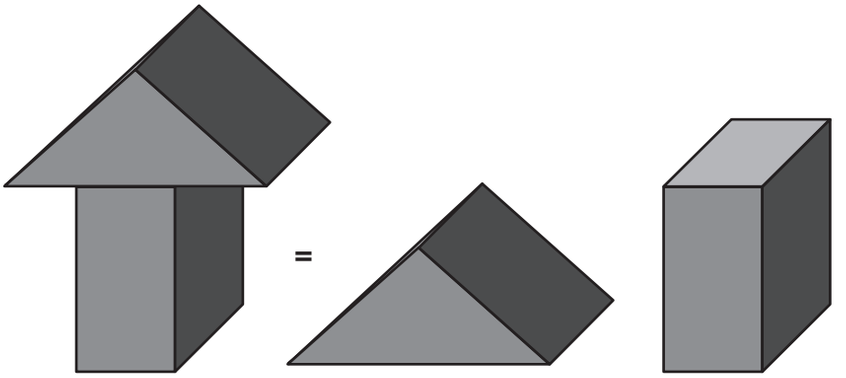
Part 2: Finding the Surface Area of Composite Objects

To find the surface area of composite objects, simply break the object up into the 3D objects that form the composite.

Consider how the shape is made from the component parts. Determine the surface area of each part. Then remove the area of any overlapping part.

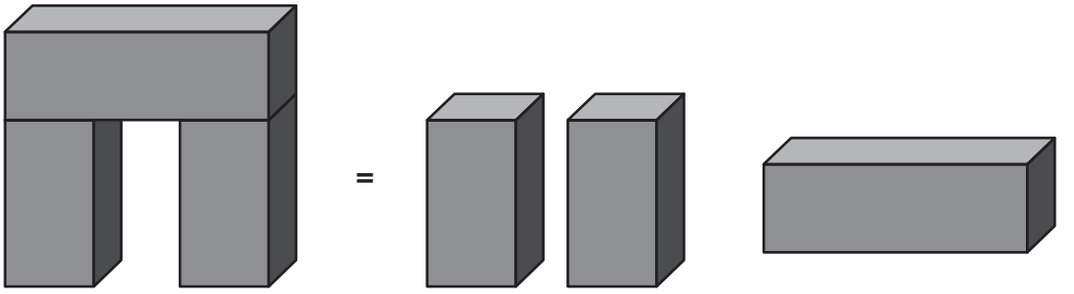
Example 1

What 3D objects make up this composite?



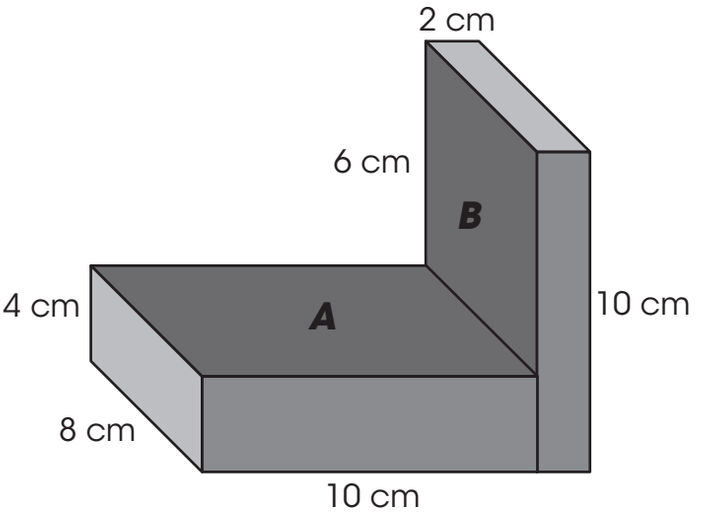
Example 2

What 3D objects make up this composite?



Example 3

What is the surface area of this composite object?



Surface Area Shape A

$SA = 2(lh) + 2(lw) + 2(hw)$
$SA = 2(4 \times 8) + 2(4 \times 10) + 2(10 \times 8)$
$SA = 2(32) + 2(40) + 2(80)$
$SA = 64 + 80 + 160$
$SA = 304 \text{ cm}^2$

Surface Area Shape B

$SA = 2(lh) + 2(lw) + 2(hw)$
$SA = 2(10 \times 2) + 2(8 \times 10) + 2(2 \times 8)$
$SA = 2(20) + 2(80) + 2(16)$
$SA = 40 + 160 + 32$
$SA = 232 \text{ cm}^2$

Overlap - This is where part A meets part B.

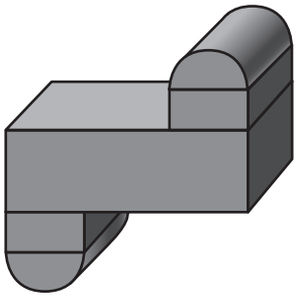
Area = lw
$A = 8 \times 4$
$A = 32 \text{ cm}^2$

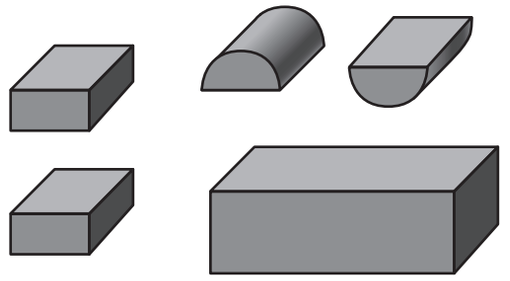
The overlap must be subtracted from Shape A and Shape B.

Total Surface Area = Area A + Area B – Overlap Area
Total Surface Area = 304 + 232 – 32 – 32
Total Surface Area = 472 cm^2

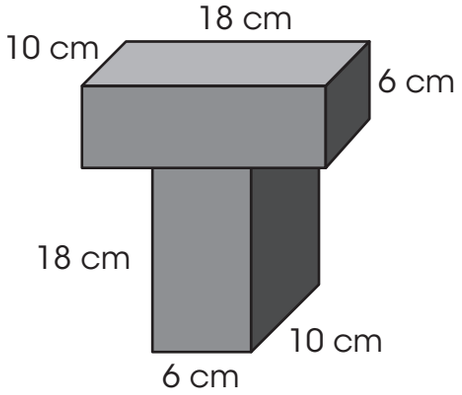
Practice Questions

1. Draw the 3D shapes that form this composite object.





2. Find the surface area of this composite object.



SA A = 2(lh) + 2(lw) + 2(hw)
SA A = 2(18 × 6) + 2(18 × 10) + 2(6 × 10)
SA A = 2(108) + 2(180) + 2(60)
SA A = 216 + 360 + 120
SA A = 696 cm²

SA B = 2(lh) + 2(lw) + 2(hw)
SA B = 2(6 × 18) + 2(6 × 10) + 2(18 × 10)
SA B = 2(108) + 2(60) +2 (180)
SA B = 216 + 120 + 360
SA B = 696 cm²

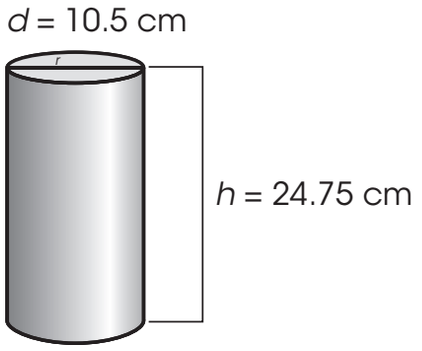
Overlap Area
A = lw
A = 6 × 10
A = 60 cm²

The overlap must be subtracted from Shape A and Shape B
Total: 696cm² + 696cm² – 60cm² – 60cm²
Total: = 1272 cm²

Lesson 10 Assignment

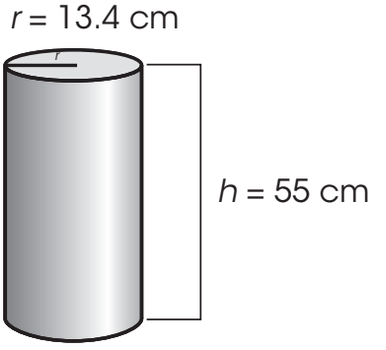
1. Find the surface area of the following cylinders.

a.



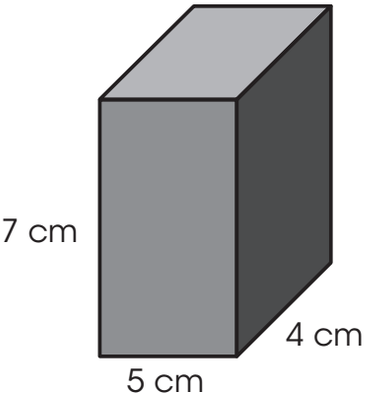
SA = 2πr² + 2πrh
SA = (2 × 3.14 × 5.25²) + (2 × 3.14 × 5.25 × 24.75)
SA = (173.0925) + (816)
Surface Area = 989.1 cm²

b.



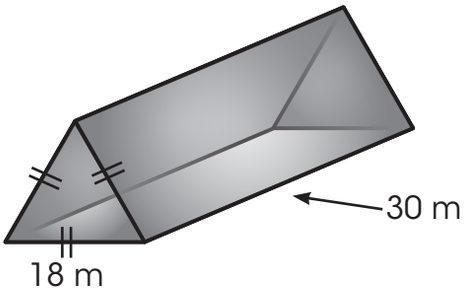
SA = 2πr² + 2πrh
SA = (2 × 3.14 × 13.4²) + (2 × 3.14 × 13.4 × 55)
SA = (1127.6368) + (4628.36)
Surface Area = 5755.9968 = 5756 cm²

2. Find the surface area for this rectangular prism.



SA = 2(lh) + 2(lw) + 2(hw)
SA = 2(5 × 7) + 2(5 × 4) + 2(7 × 4)
SA = 2(35)+ 2(20) + 2(28)
SA = 70 + 40 + 56
SA = 166 cm²

3. Find the surface area of a triangular prism.



$a^2 + b^2 = c^2$
$9^2 + b^2 = 18^2$
$81 + b^2 = 324$
$b^2 = 243$
$b^2 = 15.59\ cm$

$SA = 2\left(\frac{1}{2} \times b \times h\right) + 3\ (l \times w)$

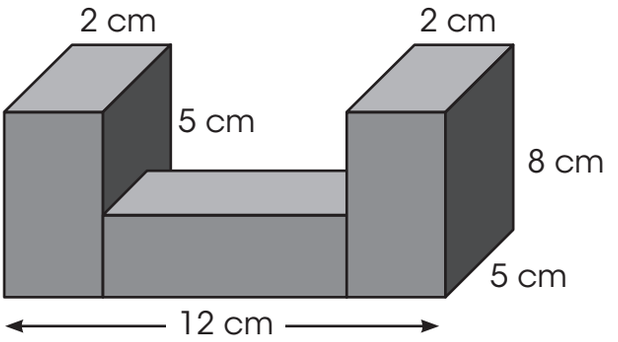
$SA = 2\left(\frac{1}{2} \times 18 \times 15.59\right) + 3\ (30 \times 18)$

$SA = 280.62 + 1620$

$SA = 1900.62\ cm^2$

4. Find the surface area of the following composite objects.

a.



SA Shape A and B

SA = 2(lh) + 2(lw) + 2(hw)
SA = 2(2 × 8) + 2(2 × 5) + 2(8 × 5)
SA = 32 + 20 + 80

Shape A SA = 132 cm²
Shape B SA = 132 cm²

SA Shape C

SA = 2(lh) + 2(lw) + 2(hw)
SA = 2(8 × 3) + 2(8 × 5) + 2 (3 × 5)
SA = 48 + 80 + 30
SA = 158 cm²

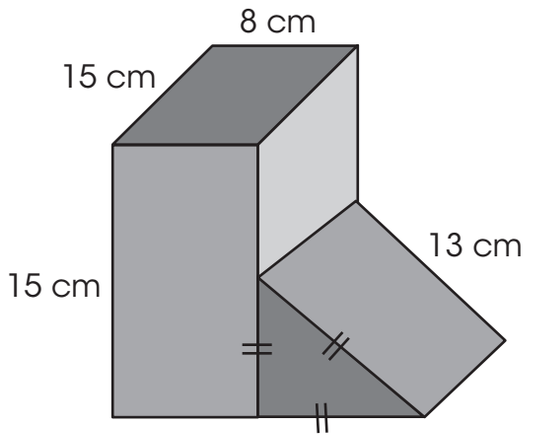
Overlap
Area = lw
A = 3 × 5
A = 15 cm²

Shape A overlap = 15 + 15 = 30 cm²
Shape B overlap = 15 + 15 = 30 cm²

Total Surface Area

SA = A + B + C – overlap
SA = 158 + 132 + 132 – 30 – 30
SA = 362 cm²

b.



SA Shape A

$SA = 2(lh) + 2(lw) + 2(hw)$
$SA = 2(8 \times 15) + 2(8 \times 15) + 2(15 \times 15)$
$SA = 240 + 240 + 450$
$SA = 930\ cm^2$

Shape B –Height of the Triangle

$a^2 + b^2 = c^2$
$6.5^2 + b^2 = 132$
$42.25 + b^2 = 169$
$b^2 = 126.75$
$b = 11.258\ cm$

SA Shape B

$SA = 2\left(\frac{1}{2} \times b \times h\right) + 3\ (l \times w)$

$SA = 2\left(\frac{1}{2} \times 13 \times 11.258\right) + 3\ (13 \times 15)$

$SA = 146.354 + 585$
$SA = 731.35\ cm^2$

Overlap
Area = lw
$A = 15 \times 13 = 195\ cm^2$ ***on each Shape***

Total SA = A + B – Overlap
$SA = 930\ cm^2 + 731.35\ cm^2 – 195\ cm^2 – 195\ cm^2$
$SA = 1271.35\ cm^2$


LEARNING
ANYTIME ANYWHERE