Important Concepts . . .

Preview Review

Mathematics Grade 9 TEACHER KEY

W3 - Lesson 12: Polygons and Scale Diagrams

Important Concepts of Grade 9 Mathematics

Materials Required

Paper
Pencil
Ruler
Protractor
Tracing Paper

No Textbook Required

This is a stand-alone course.

Mathematics Grade 9
Version 6
Preview/Review W3 - Lesson 12
ISBN: 978-1-927090-00-8

Publisher: Alberta Distance Learning Centre
Written by: Lenee Fyfe
Reviewed by: Danielle Winter

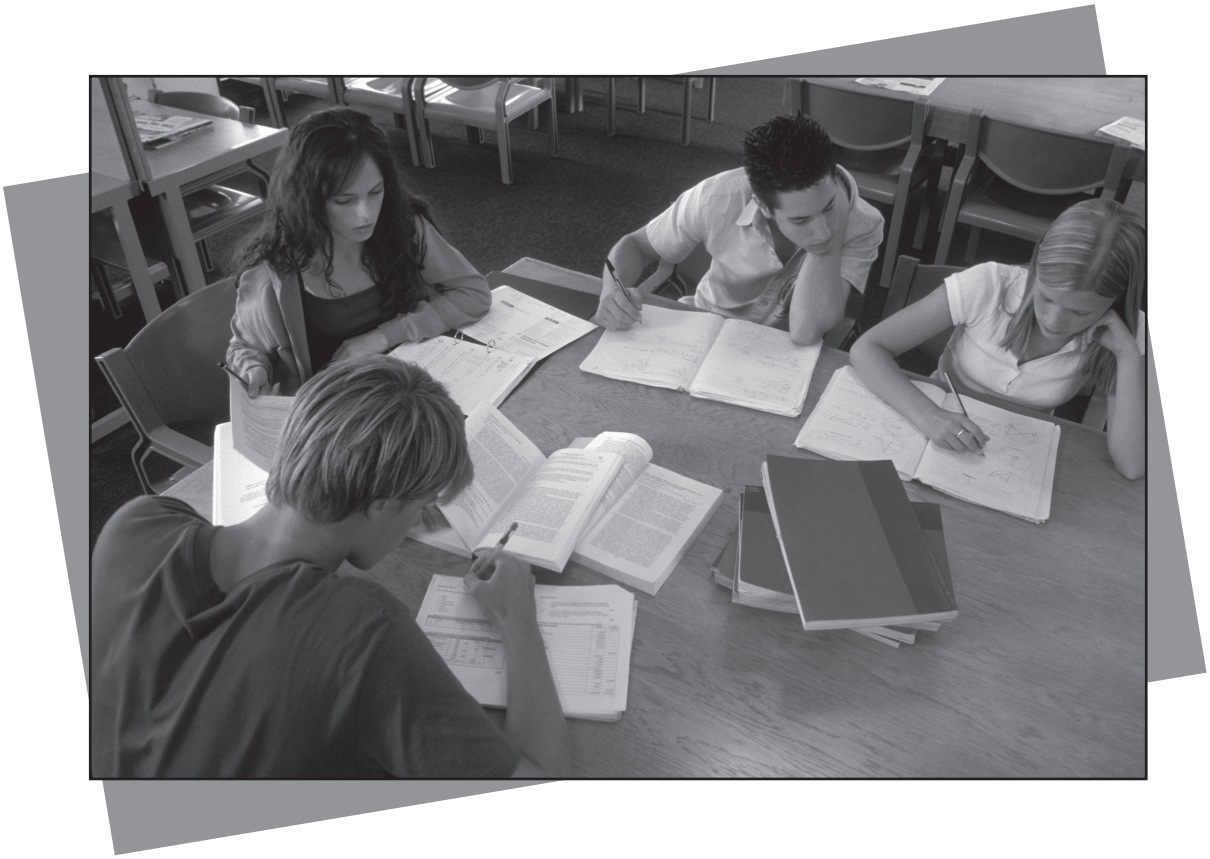
Project Coordinator: Danielle Winter
Preview/Review Publishing Coordinating Team: Julie Reschke

Alberta Distance Learning Centre has an Internet site that you may find useful. The address is as follows: **http://www.adlc.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Preview/Review Concepts for Grade Nine Mathematics

Teacher Key



W3 – Lesson 12:

Polygons and Scale Diagrams

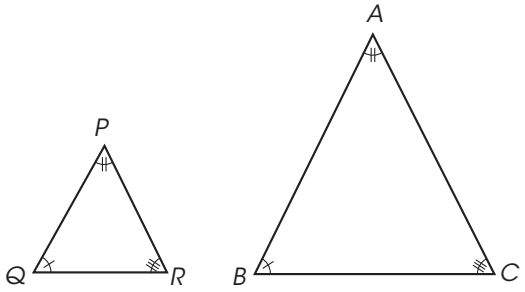
OBJECTIVES

By the end of this lesson, you will be able to:

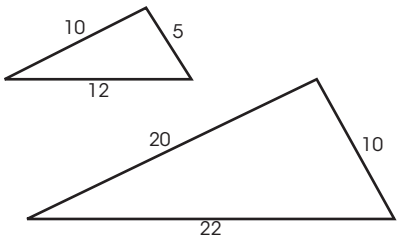
- Determine if the polygons in a given pre-sorted set are similar, and explain the reasoning.
- Solve a given problem, using the properties of similar polygons.
- Draw a diagram to scale that represents an enlargement or a reduction of a given 2-D shape.
- Determine the scale factor for a given diagram drawn to scale.
- Determine if a given diagram is proportional to the original 2-D shape, and, if it is, state the scale factor.

GLOSSARY

Corresponding Angles: Corresponding angles have the same location in geometric shapes.



Corresponding Sides: Corresponding sides have the same location in geometric shapes.



Similar Figures: These polygons have the same shape but can be different sizes. They will have the same corresponding angles and sides.

Polygon: A two-dimensional closed figure made up of three or more straight line segments.

Enlargement: To increase the dimensions of an image by a constant factor.

Reduction: To decrease the dimensions of an image by a constant factor.

Scale Factor: The constant factor that all dimensions of an image have been enlarged or reduced.

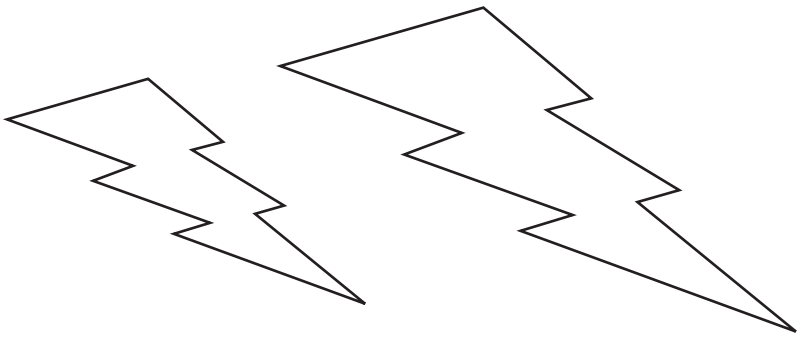
W3 – Lesson 12: Polygons and Transformations

Materials required:

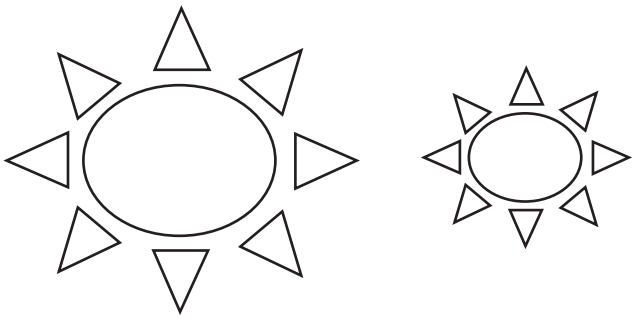
- Paper, pencil, Ruler, Protractor, and Tracing paper

Part 1: Enlargement, Reductions and Scale Factor

An ***enlargement*** is to increase an image by a constant factor. The enlargement results in a similar image. The image is the same shape but will proportionally larger than the original image.



A ***reduction*** is to decrease an image by a constant factor. The reduction results in a similar image. The image is the same shape but will be proportionally smaller than the original image.

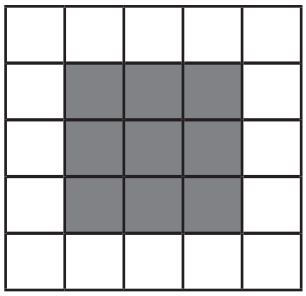


The ***scale factor*** is the constant that the dimensions of an image are multiplied by to either increase or decrease the image. If the scale factor is greater than 1, the image has been enlarged. If the scale factor is less than 1, the image has been reduced.

The ***scale diagram*** is a larger or smaller representation of the image that has been proportionally enlarged or reduced.

Example 1

Draw the image with a scale factor of 2.

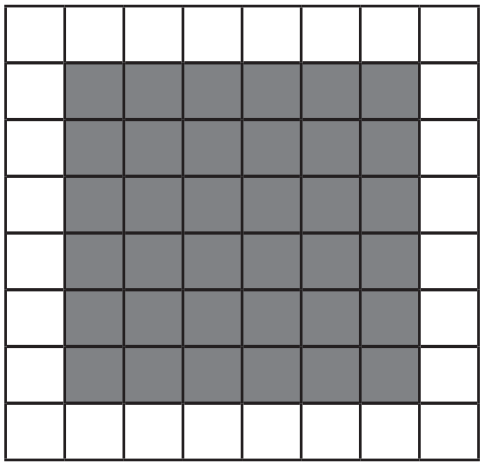


Multiply each side of the original shape by 2. Remember, scale factors that are greater than 1 indicate the image has been enlarged.

3 units × 2 = 6
3 units × 2 = 6

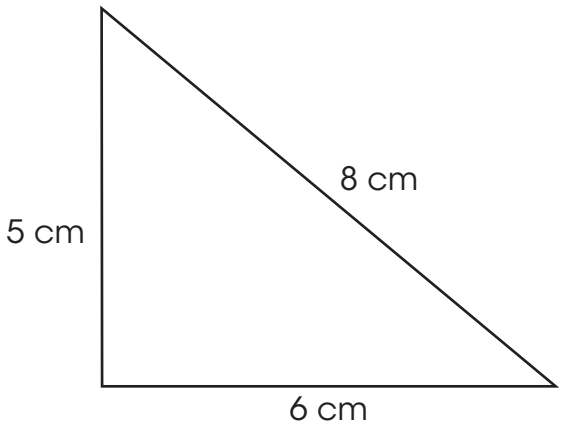
The new dimensions are:
6 units and 6 units.

The enlarged image should have new dimensions of 6 × 6. The scale factor is 2.



Example 2

Draw the image with a scale factor of 0.5.

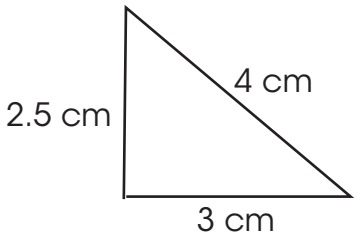


Multiply each side of the original shape by 0.5. Remember, scale factors that are less than 1 indicate the image has been reduced.

8 cm × 0.5 = 4 cm
5 cm × 0.5 = 2.5 cm
6 cm × 0.5 = 3 cm

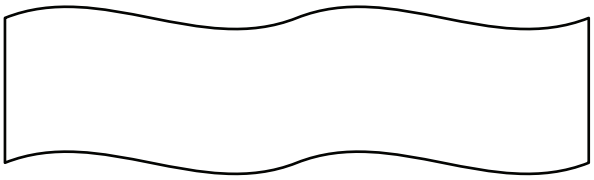
The new dimensions are: 2.5, 4 and 3.

Draw the image with a scale factor of 0.5.



Example 3

Below is a drawing of a banner used to advertise a garage sale. The scale of the banner is 1:3. If the drawing of the banner below measures 144 cm, what is the actual length of the banner?



To solve for the actual length of the banner, set up a proportion, using the scale factor.

$$\frac{1}{3} = \frac{144}{x}$$

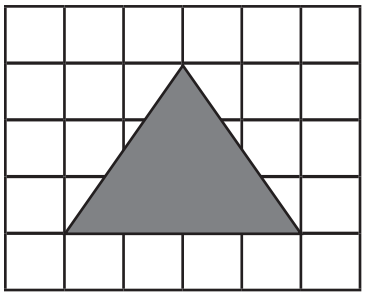
Solve for the unknown value x.

$$\frac{1}{3} = \frac{144}{x}$$
$$\frac{1}{3}x = 144$$
$$x = (144)(3)$$
$$x = 432 \text{ cm}$$

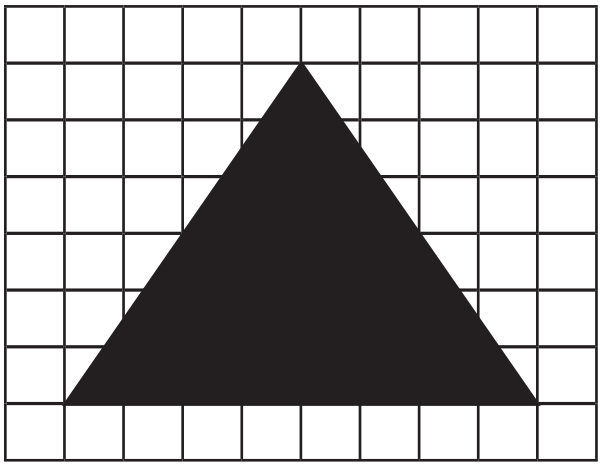
The actual length of the banner is 432 cm.

Practice Questions

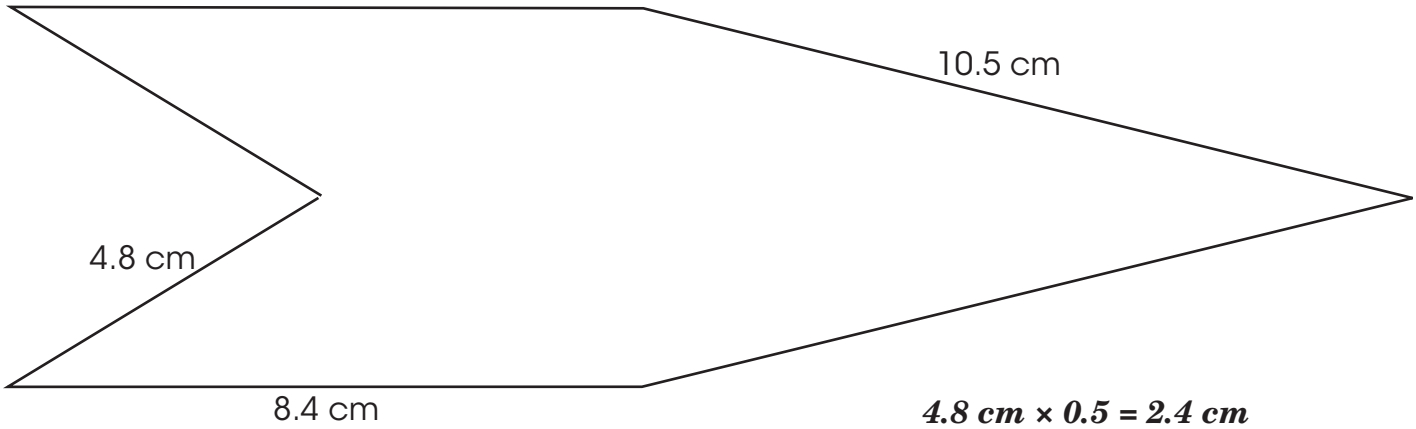
1. Draw the image with a scale factor of 2.



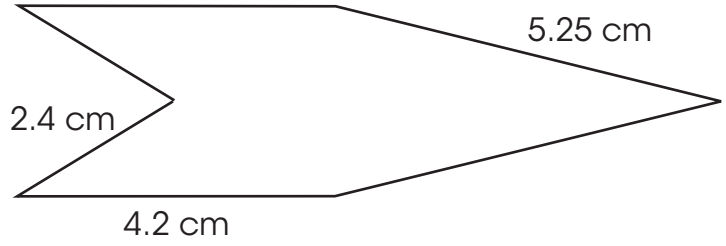
L = 4 units × 2 = 8
H = 3 units × 2 = 6



2. Draw the image with a scale factor of 0.5.



4.8 cm × 0.5 = 2.4 cm
8.4 cm × 0.5 = 4.2 cm
10.5 cm × 0.5 = 5.25 cm



3. Determine the correct length for the given scale factor.

   a. A poster measures 40 cm × 60 cm. What are the dimensions of the scale diagram with a scale factor of 0.5?

   40 × 0.5 = 20 cm
   60 × 0.5 = 30 cm

   b. A skateboard has wheels with a diameter of 54 mm. What are the dimensions of the scale diagram with a scale factor of 3?

   54 × 3 = 162 mm

   c. A table has a length of 90 cm. What are the dimensions of the scale diagram with a scale factor of $\frac{1}{9}$?

   $$90 \times \frac{1}{9} = 10 \text{ cm}$$

4. Solve the following proportions.

   a. $90 = \frac{x}{117}$ b. $\frac{1}{12} = \frac{10.5}{x}$ c. $\frac{1}{x} = \frac{3}{156}$

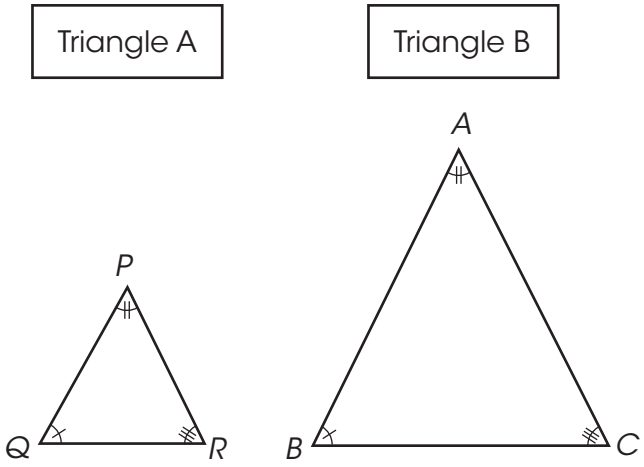
   x = 10 530 ***x = 126*** ***x = 52***

Part 2: Exploring Triangles

Similar polygons may be of different sizes or orientation, but they must be the same shape. To see if polygons are similar, check to see if they have corresponding sides and corresponding angles.

Corresponding sides are sides in polygons that have the same relative position in figures.

Corresponding angles are angles in polygons that have the same relative position in figures.



Corresponding Sides

	Triangle A	Triangle B
Line	PQ	AB
Line	PR	AC
Line	QR	BC

Corresponding Angles

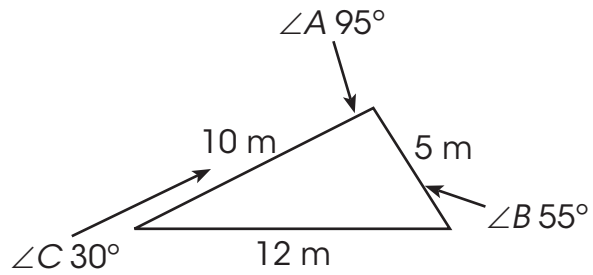
	Triangle A	Triangle B
Angle	$\angle P$	$\angle A$
Angle	$\angle Q$	$\angle B$
Angle	$\angle R$	$\angle C$

Once triangles have been determined to be similar, problems related to similar triangles can be solved using either scale factor or proportion.

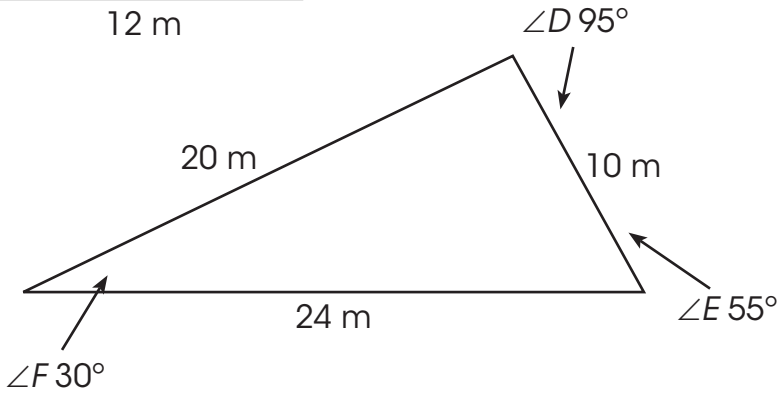
Example 1

Decide if these triangles are similar.

Triangle A



Triangle B



To see if the triangles are similar, check to see if they have either corresponding angles or corresponding sides.

$\angle A = 95°$ and $\angle D = 95°$
$\angle B = 55°$ and $\angle E = 55°$
$\angle C = 30°$ and $\angle F = 30°$

The corresponding angles are all equal.

Now check to see if there are corresponding sides. In order for the sides to correspond, the ratio of each side must be equal.

Triangle A sides lengths = 10 m, 12 m, 5 m
Triangle B side lengths = 20 m, 24 m, 10 m

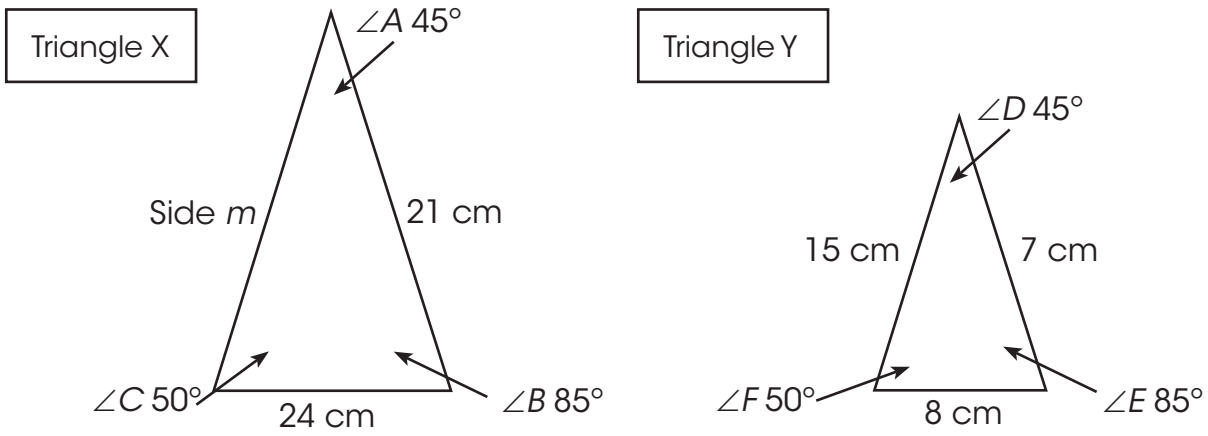
The ratio of Triangle B to Triangle A.

$\frac{20}{10} = 2$ $\frac{24}{12} = 2$ $\frac{10}{5} = 2$

All sides are proportional with a scale factor of 2. So Triangle A and Triangle B are proportional.

Example 2

The following triangles below are similar. Determine the length of the missing side m.



To find the missing side, compare the corresponding sides to determine the scale factor. Once the scale factor is known, solve for the missing side.

$$\frac{AC}{DF} = \frac{m}{15} \qquad \frac{CB}{FE} = \frac{24}{8} \qquad \frac{BA}{ED} = \frac{21}{7}$$
$$= 3 \qquad = 3$$

The scale factor is 3.

Using algebra, $\frac{m}{15} = 3$

$$\frac{m}{15} \times 15 = 3 \times 15$$
$$m = 45$$

Or solve by using proportion, $\frac{AC}{DF} = \frac{CB}{FE}$

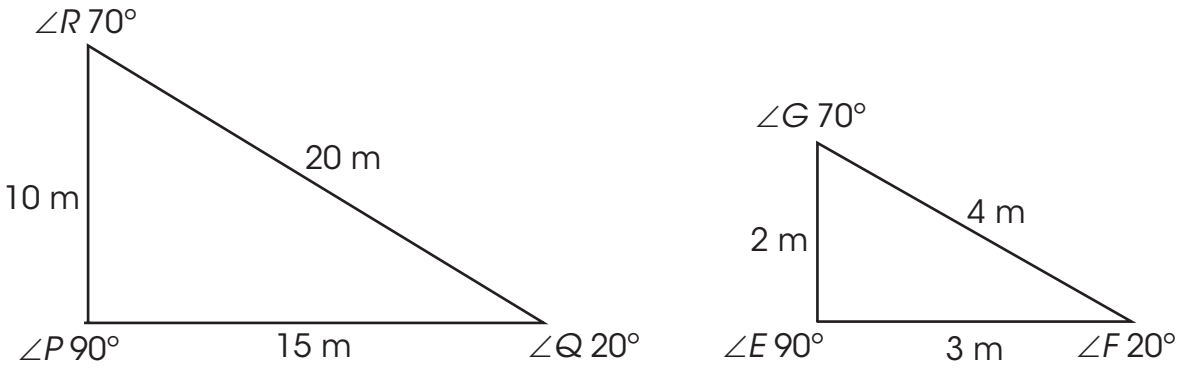
$$\frac{m}{15} = \frac{24}{8} \quad \text{Cross Multiply}$$
$$8m = 360$$
$$\frac{8m}{8} = \frac{360}{8}$$
$$m = 45$$

Using either method, $m = 45$. The missing side is 45 cm.

Practice Questions

1. Determine if the following triangles are similar by comparing both the sides and the angles.

a.



∠P = 90° and ∠E = 90° ∠Q = 20° and ∠F = 20° ∠R = 70° and ∠G = 70°

The corresponding angles are all equal. To see if there are corresponding sides, the ratio of each side must be equal.

Triangle A sides lengths = 15 m, 20 m, 10 m
Triangle B side lengths = 3 m, 4 m, 2 m

The ratio of triangle B to triangle A

$$\frac{15}{3} = 5 \qquad \frac{20}{4} = 5 \qquad \frac{10}{2} = 5$$

Triangle A and triangle B are proportional, with a scale factor of 5.

b. Use the chart below to determine if Triangle A and B are similar.

Corresponding Angles

	Triangle A	Triangle B
Angle *X*	∠85°	∠85°
Angle *Y*	∠65°	∠65°
Angle *Z*	∠30°	∠30°

Corresponding Sides

	Triangle A	Triangle B
Line	45 m	15 m
Line	27 m	9 m
Line	24 m	6 m

The corresponding angles are all equal. Now check to see if the sides correspond (the ratio of each side must be equal).

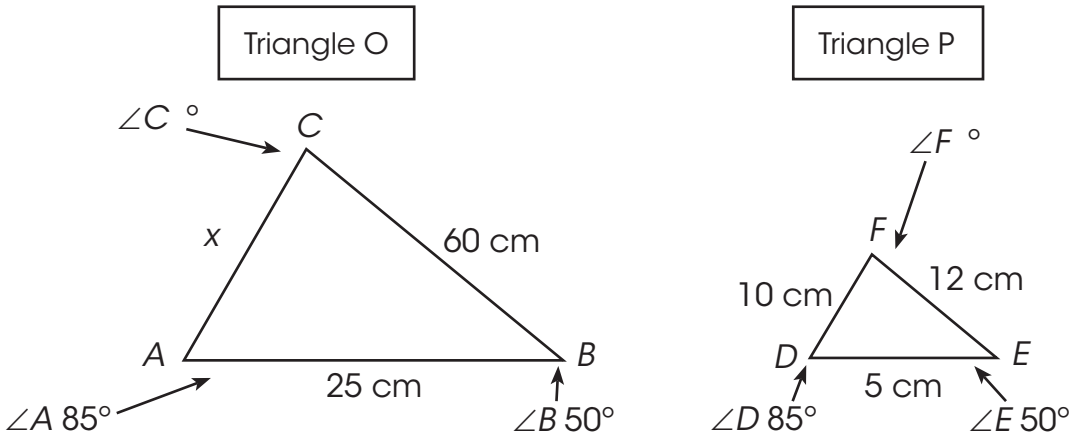
Triangle A sides lengths = 45 m, 27 m, 24 m
Triangle B side lengths = 15 m, 9 m, 6 m

The ratio of triangle A to triangle B

$$\frac{45}{15} = 3 \qquad \frac{27}{9} = 3 \qquad \frac{24}{6} = 4$$

Only 2 of the sides have a scale factor of 3. The remaining side has a scale factor of 4. This is not proportional so these triangles are not similar.

2. The following triangles are similar. Find the missing side and angles.



$\angle C$ ***45°***

$\angle F$ ***45°***

x ***50 cm***

Since these triangles are similar:
$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.
Using triangle P, all the angles should have a sum of 180°.
So, $50° + 85° + \angle F = 180°$
$\angle F = 45°$ and $\angle C = 45°$ because the triangles are similar.

To find the missing side, find the scale factor.
$\frac{CB}{FE} = \frac{60}{12} = 5$ ***and*** $\frac{AB}{DE} = \frac{25}{5} = 5$***, so the scale factor is 5.***

Using the scale factor,

$\frac{x}{10} = 5$

$x = (5)(10)$

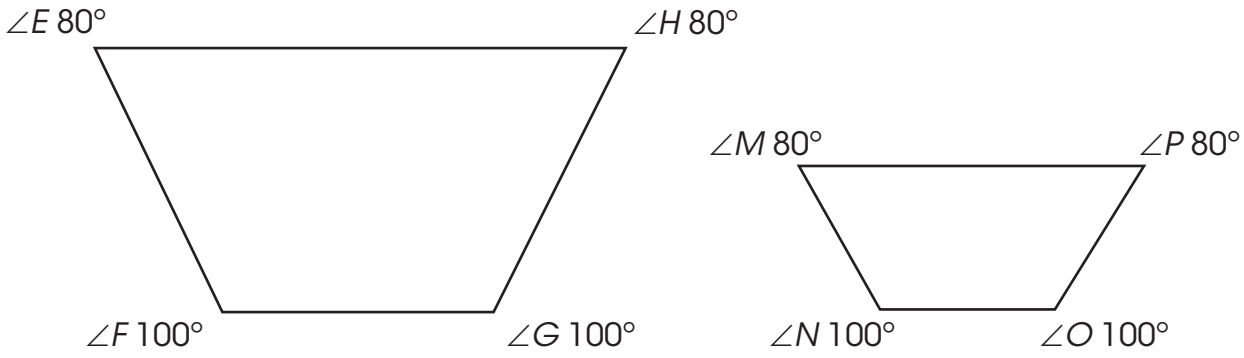
$x = 50$ ***cm***

Part 3: Exploring Polygons

A polygon is a two dimensional figure that is made up of three or more line segments. These line segments are straight. Polygons do not have any curved edges.

To determine if polygons are similar, like triangles, they must have corresponding angles that are equal in measure. They also must have side lengths that are proportional.

Like triangles, similar polygons can be used to determine unknown side lengths or angle measures.



Corresponding Sides

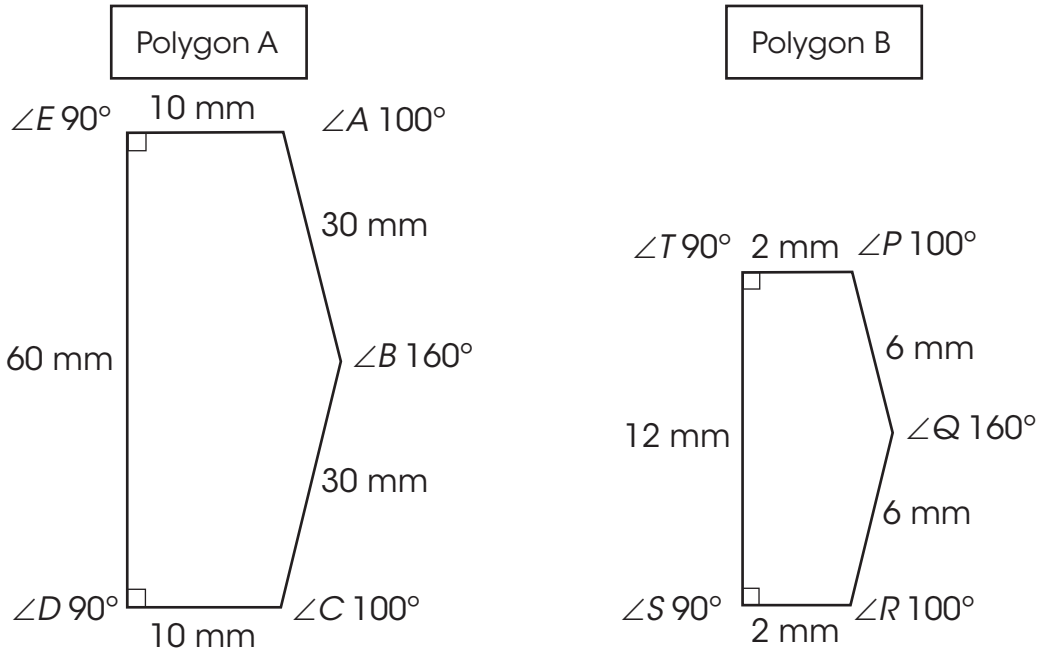
	Polygon A	Polygon B
Line	*EF*	*MN*
Line	*FG*	*NO*
Line	*GH*	*OP*
Line	*HE*	*PM*

Corresponding Angles

	Triangle A	Triangle B
Angle	∠*E*	∠*M*
Angle	∠*F*	∠*N*
Angle	∠*G*	∠*O*
Angle	∠*H*	∠*P*

Example 1

Decide if these polygons are similar.



To see if the polygons are similar, check to see if they have either corresponding angles or corresponding sides.

$\angle A = 100°$ and $\angle$P = 100° $\angle B = 160°$ and $\angle Q = 160°$
$\angle C = 100°$ and $\angle R = 100°$ $\angle D = 90°$and $\angle S = 90°$
$\angle E = 90°$ and $\angle T = 90°$

The corresponding angles are all equal. Now check to see if there are corresponding sides.

Polygon A sides lengths = 60 mm, 10 mm, 30 mm, 30 mm, 10 mm
Polygon B side lengths = 12 mm, 2 mm, 6 mm, 6 mm, 2 mm

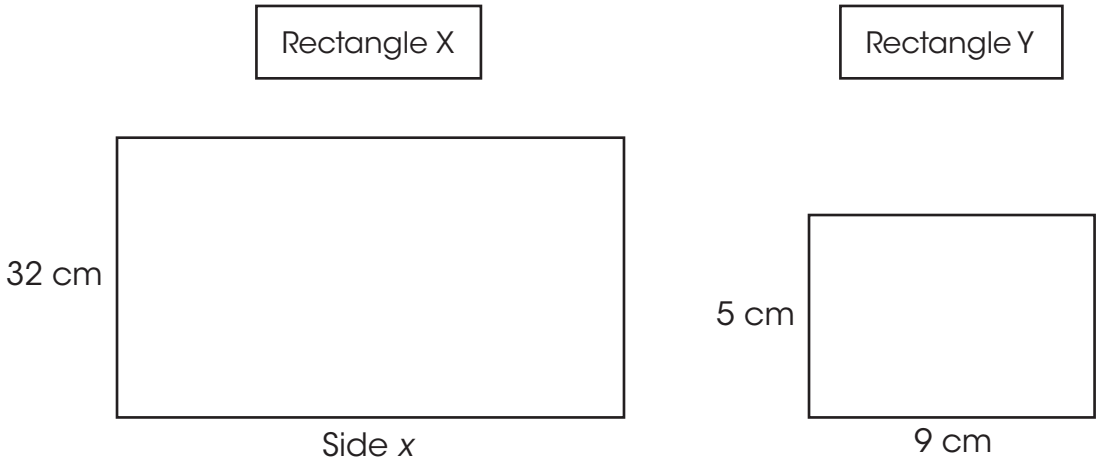
The ratio of Polygon A to Polygon B.

$$\frac{60}{12} = 5 \quad \frac{10}{2} = 5 \quad \frac{30}{6} = 5 \quad \frac{30}{6} = 5 \quad \frac{10}{2} = 5$$

All sides are proportional with a scale factor of 5. So Polygon A and Polygon B are proportional.

Example 2

The following rectangles below are similar. Determine the length of the missing side x.



Since the rectangles are similar, the side lengths will be proportional. Set up a proportion to solve for the unknown side.

$\frac{32}{5} = \frac{x}{9}$ Cross Multiply

$5x = 288$

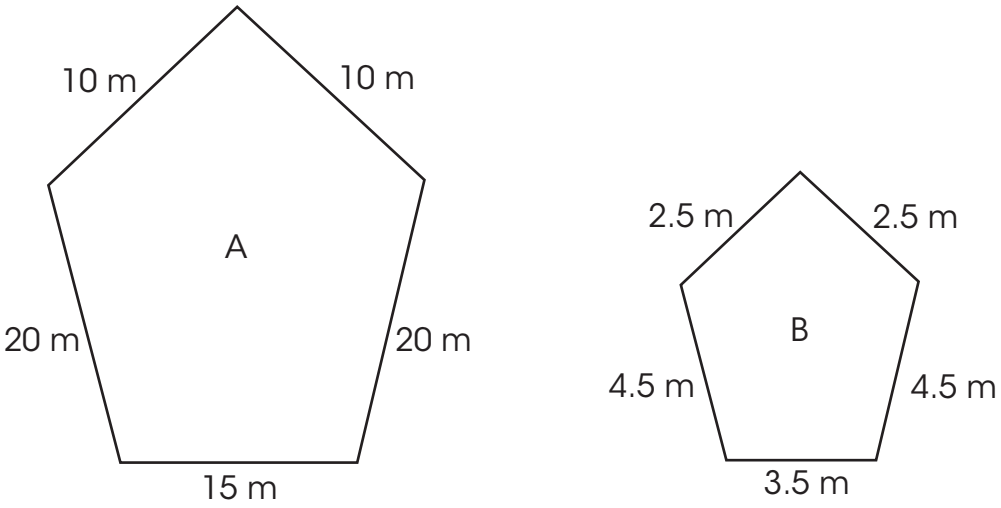
$\frac{5x}{5} = \frac{288}{5}$

$x = 57.6$

The missing side length on Rectangle X is 57.6 cm.

Practice Questions

1. Determine if the following polygons are similar by comparing both the sides and the angles.



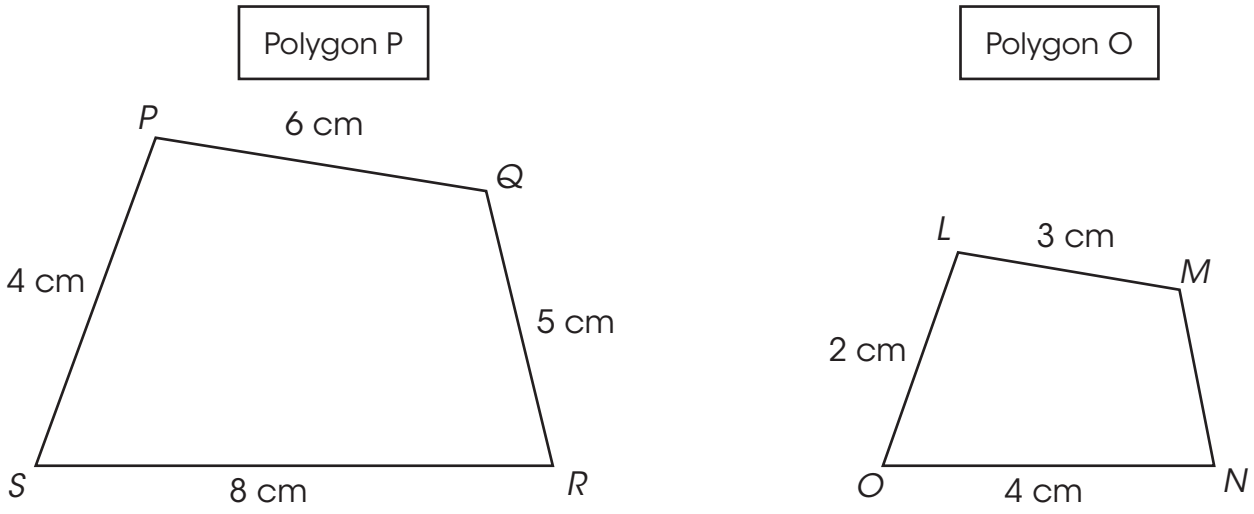
Polygon A sides lengths = 10 m, 20 m, 15 m, 20 m, 10 m
Polygon B side lengths = 2.5 m, 4.5 m, 3.5 m, 4.5 m, 2.5 m

The ratio of polygon A to polygon B

$$\frac{10}{2.5} = 4 \quad \frac{20}{4.5} = 4.44... \quad \frac{15}{3.5} = 4.2875... \quad \frac{20}{4.5} = 4.44... \quad \frac{10}{2.5} = 4$$

All sides are not proportionate. The proportions range from 4 – 4.444...
Since the side are not proportional, these polygons are not similar.

2. The following polygons are similar. Find the length of the missing side.



To find the missing side, set up a proportion.

$$\frac{PQ}{QR} = \frac{LM}{MN}$$

$$\frac{6}{5} = \frac{3}{mn} \qquad \textit{Cross Multiply}$$

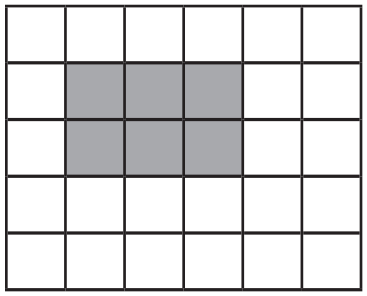
$$6mn = 15$$

$$6mn = 2.5$$

The missing side is 2.5 cm.

Lesson 12 Assignment

1. a. Draw the image with a scale factor of 3.

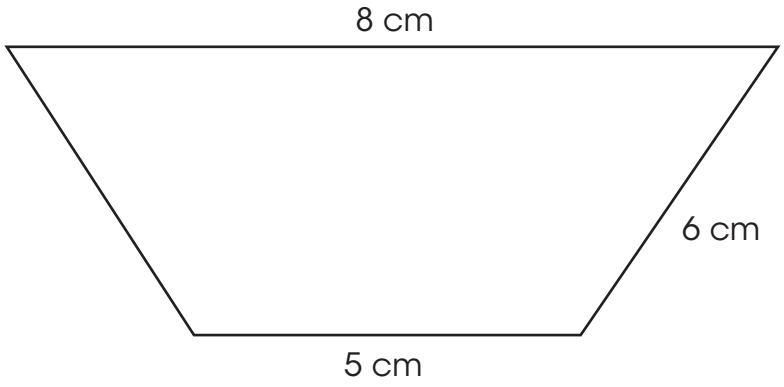


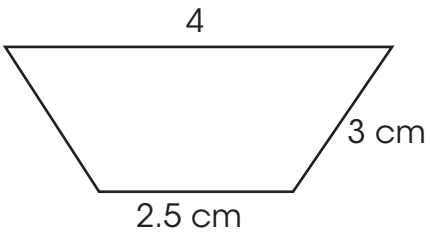
L = 3 units × 3 = 9 units
H = 2 units × 3 = 6 units

The enlarged image should have new dimensions of 9 × 6. The scale factor is 3 units.

b. Is this a reduction or an enlargement? ***an enlargement***

2. a. Draw the image with a scale factor of 0.5.





The new dimensions are:

5 cm × 0.5 = 2.5 cm
6 cm × 0.5 = 3 cm
8 cm × 0.5 = 4 cm

b. Is this a reduction or an enlargement? ***a reduction***

3. Determine the correct length for the given scale factor.

   a. A board game measures 50 cm × 80 cm. What are the dimensions of the scale diagram with a scale factor of 4?

   50 × 4 = 200 cm
   80 × 4 = 320 cm

   b. A fan has a diameter of 54 mm. What are the dimensions of the scale diagram with a scale factor of $\frac{1}{3}$?

   $$54 \times \frac{1}{3} = 18 \text{ mm}$$

   c. A desk has a length of 1.5 m. What are the dimensions of the scale diagram with a scale factor of 3?

   1.5 × 3 = 4.5 m

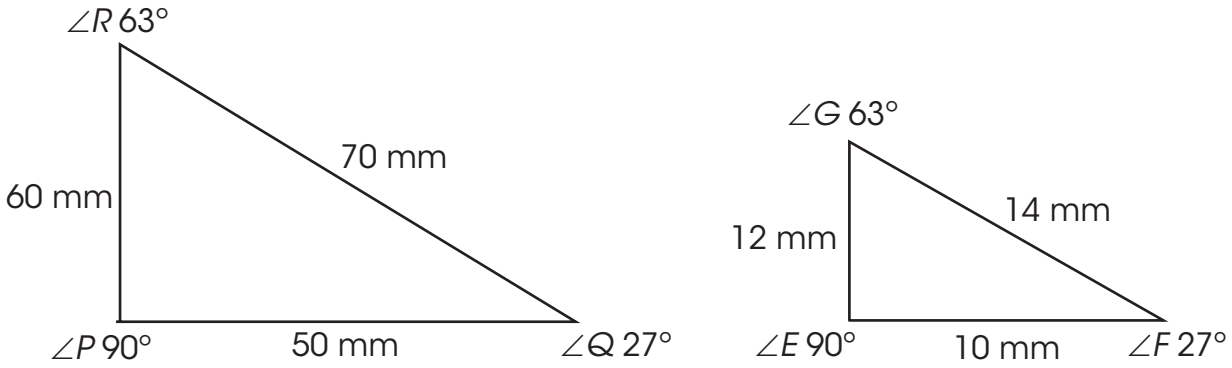
4. Solve the following proportions. Round to the nearest tenth when needed.

   a. $\frac{1}{7} = \frac{x}{249.2}$ ***x = 35.6***

   b. $\frac{1}{15} = \frac{12.8}{x}$ ***x = 192***

   c. $\frac{1}{x} = \frac{18}{278}$ ***x = 15.4***

5. a. Determine if the following triangles are similar by comparing both the sides and the angles.



∠P = 90° and ∠E = 90° ∠Q = 27° and ∠F = 27° ∠R = 63° and ∠G = 63°

Triangle A sides lengths = 50 mm, 60 mm, 70 mm
Triangle B side lengths = 10 mm, 12 mm, 14 mm

The ratio of triangle A to triangle B

$$\frac{50}{10} = 5 \qquad \frac{60}{12} = 5 \qquad \frac{70}{14} = 5$$

All sides are proportional with a scale factor of 5. So Triangle A and Triangle B are proportional.

b. Use the chart below to determine if Triangle A and B are similar.

Corresponding Angles

	Triangle A	Triangle B
Angle *X*	∠60°	∠60°
Angle *Y*	∠25°	∠25°
Angle *Z*	∠105°	∠105°

Corresponding Sides

	Triangle A	Triangle B
Line	105 mm	35 mm
Line	90 mm	30 mm
Line	81 mm	9 mm

∠X = 60° and ∠x = 60° ∠Y = 25° and ∠y = 25° ∠Z = 105° and ∠z = 105°

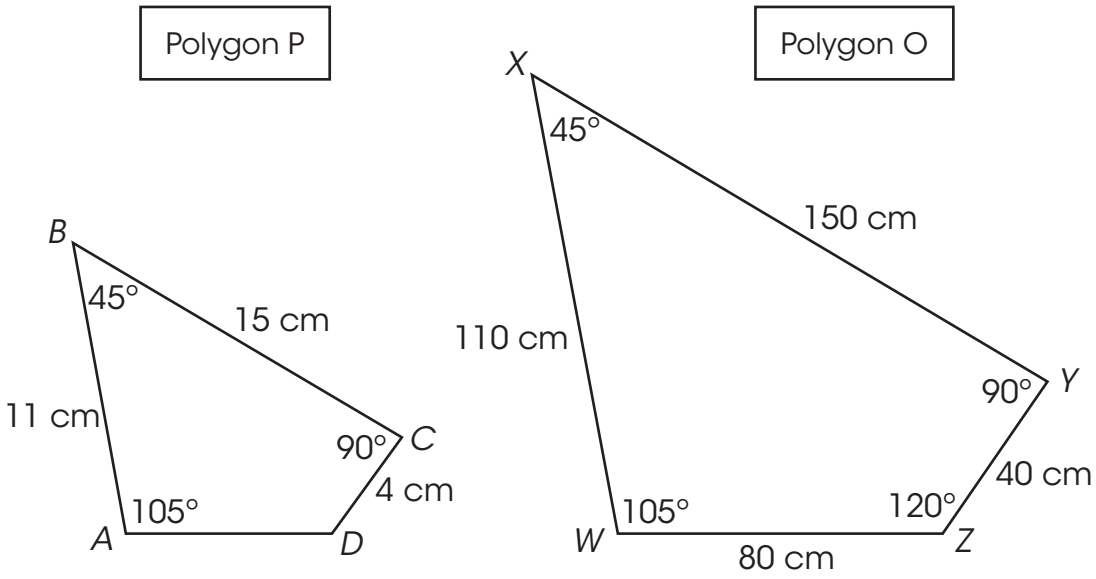
Triangle A sides lengths = 105 mm, 90 mm, 81 mm
Triangle B side lengths = 35 mm, 30 mm, 9 mm

The ratio of triangle A to triangle B

$$\frac{105}{35} = 3 \qquad \frac{90}{30} = 3 \qquad \frac{81}{9} = 9$$

Only 2 of the sides have a scale factor of 3. The remaining side has a scale factor of 9. This is not proportional so these triangles are not similar.

6. The following polygons are similar. Find the length of the missing side *AD*.



Since these polygons are similar to find the missing side, set up a proportion.

$$\frac{AD}{WZ} = \frac{BC}{XY}$$
$$\frac{AD}{80} = \frac{15}{150}$$
$$\frac{AD}{80} = 0.1$$
$$AD = (80)(0.1)$$
$$AD = 8$$


LEARNING
ANYTIME
ANYWHERE