Important Concepts . . .

Preview Review

Mathematics Grade 9 TEACHER KEY

W2 - Lesson 9: Polynomials

Important Concepts of Grade 9 Mathematics

Materials Required

Paper
Pencil
Graph Paper
Calculator

No Textbook Required

This is a stand-alone course.

Mathematics Grade 9
Version 6
Preview/Review W2 - Lesson 9
ISBN: 978-1-927090-00-8

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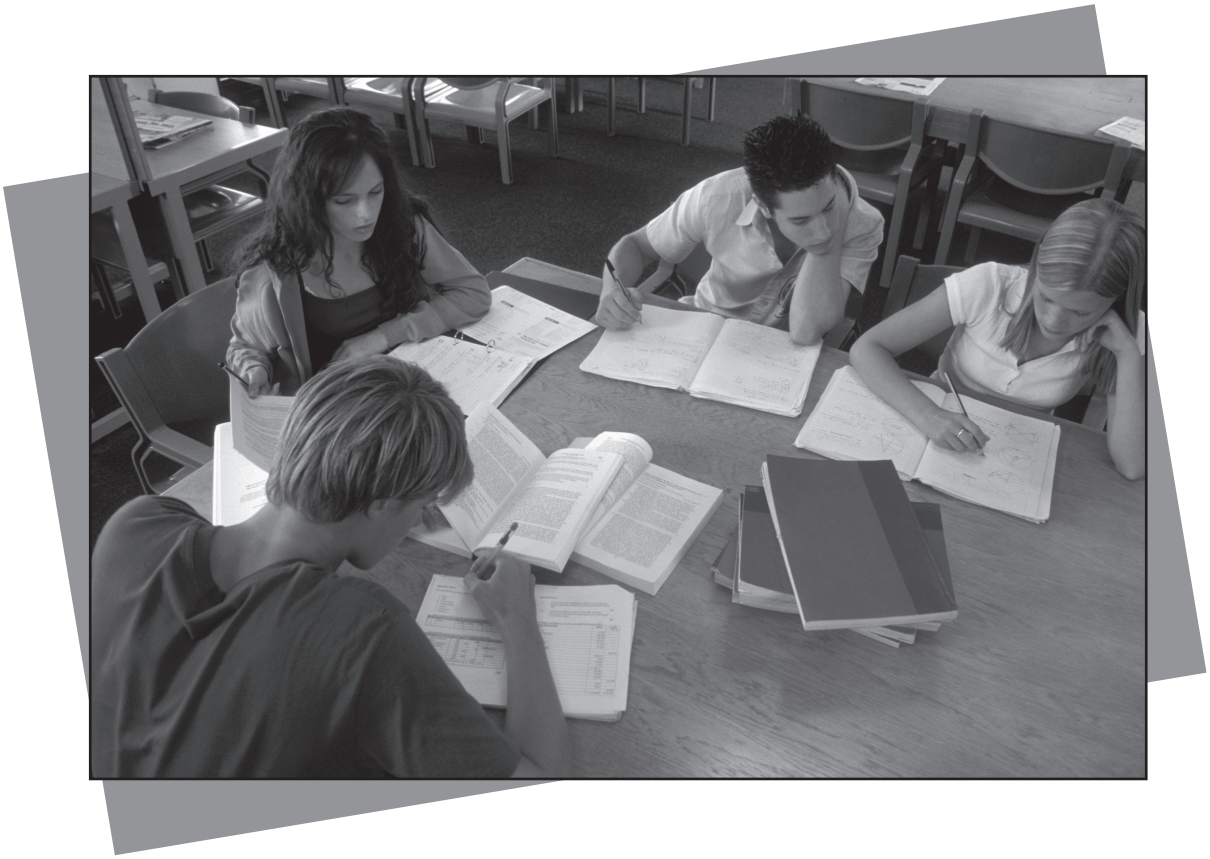
Preview/Review Concepts for Grade Nine Mathematics

Teacher Key



W2 – Lesson 9:

Polynomials

OBJECTIVES

By the end of this lesson, you will be able to:

- Model and solve problems using linear equations.
- Create a concrete model or a pictorial representation for a given polynomial expression.
- Write the expression for a given model of a polynomial.
- Identify the variables, degree, number of terms and coefficients, including the constant term, of a given simplified polynomial expression.
- Describe a situation for a given first degree polynomial expression.
- Match equivalent polynomial expressions given in simplified form; e.g., $4x - 3x^2 + 2$ is equivalent to $-3x^2 + 4x + 2$.
- Model addition/subtraction/multiplication/division of two given polynomial expressions concretely or pictorially, and record the process symbolically.

GLOSSARY

Term: A grouping of numbers and/or a variables in an algebraic equation or expression. Terms are often separated by functions like addition or subtraction.

Example: $4x^2 + 3$ The terms are $4x^2$ and 3. Note, these are connected by an addition operation.

Polynomial: An algebraic expression or equation made up of more than one term. These terms are connected together by addition and subtraction operations.

Degree of a Term: The highest-degree term in the polynomial. The degree of a term is the sum of the exponents on the variables in a single term.

Like Terms: Terms that differ only by their numerical coefficient.

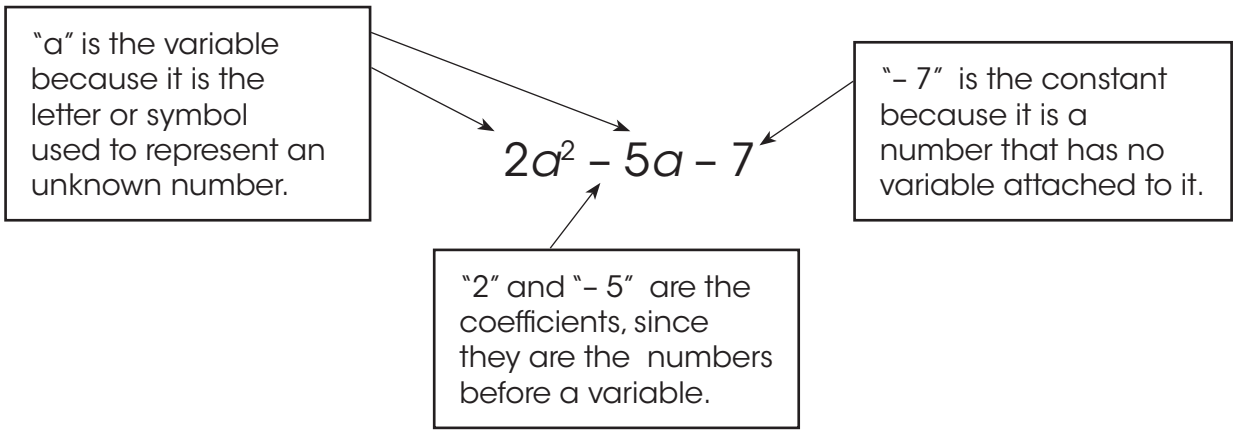
Opposite Integers: Polynomials that add together to make zero.

W2 – Lesson 9: Polynomials

Materials required:

- Paper, Pencil, Graph Paper, and Calculator

Part 1: Parts of a Polynomial



Part 2: Modelling Polynomials

A polynomial is an algebraic expression or equation that is made up of terms (a group of numbers and/or a variable). These terms are connected together by addition or subtraction operations.

$-mnz$	has 1 term	An algebraic expression with 1 term is a **monomial**.
$9y^2 + 6$	has 2 terms	An algebraic expression with 2 terms is a **binomial**.
$6d^2 + 4cd - c^2$	has 3 terms	An algebraic expression with 3 terms and is a **trinomial**.

Example 1 – The Degree of a Polynomial

To determine the degree of a polynomial with only one variable, the greatest exponent will be the degree.

$-16y^2 - 5y$	The greatest exponent in this binomial is 2. Therefore, the degree is 2.
$7x^2 + 5n - 8$	This polynomial has three terms. This is called a trinomial. $7x^2$ is the term with the highest degree. The degree is 2. Therefore, the degree of the polynomial is 2.

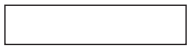
Example 2 – Modelling Polynomials

Polynomials can be modelled using algebra tiles.

is a positive 1 tile.

is a negative 1 tile.

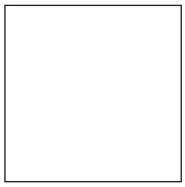
A 1 tile has the dimensions that are 1 unit by 1 unit.



is a positive x tile

is a negative x tile

An ***x*** tile has the dimensions that are ***x*** units by 1 unit.



is a positive x^2 tile

An x^2 tile has the dimensions that are ***x*** units by x units.

is a negative x^2 tile

Practice Questions

1. For each expression, complete the chart.

Expression	Number of Terms	Variable(s)	Coefficient(s)	Constant(s)	Type of Polynomial
$3x^2 + 5x - 7$	***3***	***x***	***3, 5***	***–7***	***Trinomial***
$g^2 - f^2$	***2***	***g, f***	***1, –1***	***none***	***Binomial***
$-11h$	***1***	***H***	***–11***	***none***	***Monomial***
–5	***1***	***none***	***none***	***–5***	***Monomial***

2. Write an algebraic expression for each of the following.

   a. The product of 6 and g

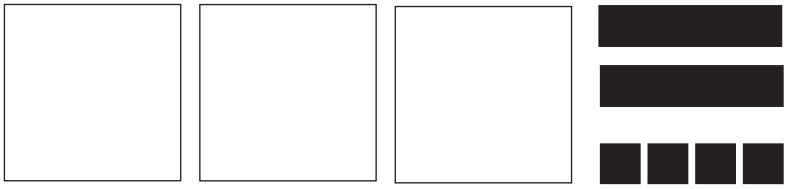
   6g

   b. The sum of $5x$ and 4

   5x + 4

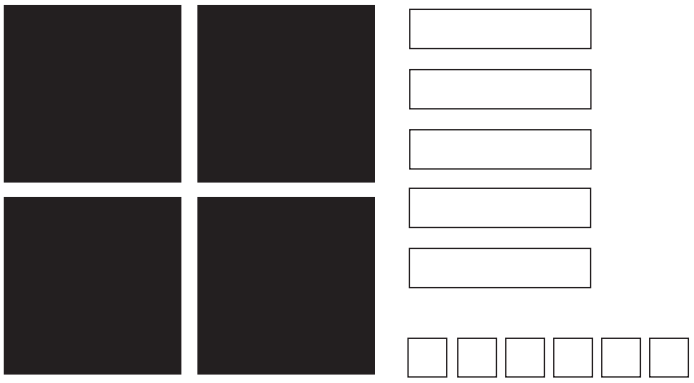
3. Write the expression for each of the following modelled expressions.

   a.



   $3x^2 - 2x - 4$

   b.



   $-4x^2 + 5x + 6$

Part 2: Adding Polynomials

To add and subtract polynomials, it is important to first combine like terms. Like terms differ only by their numerical coefficients.

Examples of like terms are:

- $3x$ and $-4x$ $6y^2$ and $-8y^2$ $-7xy$ and xy

Combining like terms will simplify the polynomial. For example:

$4x - 2x + x^2 - 5 + 5x^2 + 2$

The terms are:	$4x$ and $2x$	simplifies to $6x$
	x^2 and $5x^2$	simplifies to $6x^2$
	-5 and $+2$	simplifies to -3

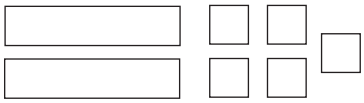
The simplified expression is $6x^2 + 6x - 3$

Example 1 – Adding Polynomials

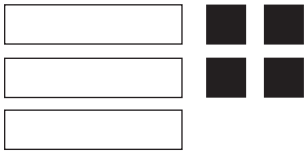
To add **$(2x + 5)$ and $(3x - 4)$**, first simplify the answer by collecting like terms. The sum of these polynomials can be found by using symbols or by using models.

$(2x + 5) + (3x - 4)$	Write out the polynomial.
$= 2x + 5 + 3x - 4$	Remove the brackets.
$= 5x + 5 - 4$	Collect the "x" terms.
$= 5x + 1$	Collect the numerical terms (constants).

To model **$2x + 5$ and $3x - 4$**, draw algebra tiles.

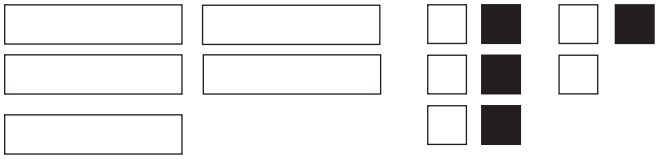


$(2x + 5)$

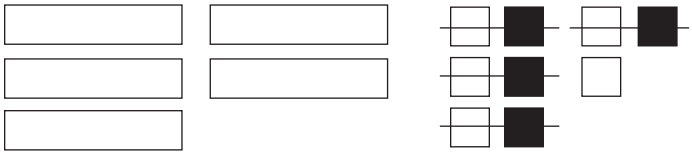


$(3x - 4)$

Collect like tiles.

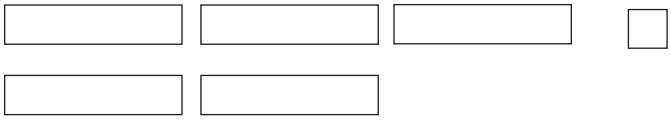


Pair off opposite tiles, which cancel each other out.



The tiles have been paired to make zeros.

Model the tiles that remain after terms have been collected and zeros have been considered.



Write the expression, represented by the tiles:

$= 5x + 1$

Example 2 – Subtracting Polynomials

When finding the difference in polynomials, determine the opposite polynomials. This is the polynomial that adds to zero. This is found by finding the opposite term of each term in polynomial expression.

For example, to find the opposite of $x^2 - 1$ find the opposite of each term.

x^2	The opposite is	$-x^2$
-1	The opposite is	$+1$

So the opposite polynomial of $x^2 - 1$ is: $-x^2 + 1$

This strategy is used to subtract polynomials.

Solve: $(3x - 4) - (2x + 3)$

The first step is to add the opposites. So the above becomes:

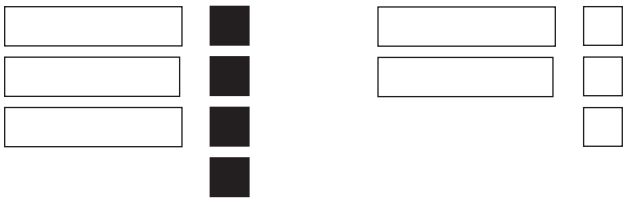
$(3x - 4) + (-2x - 3)$

> Notice the $2x$ became $-2x$. The 3 became -3. These are opposites. Opposites are used in only the numbers you are subtracting (ie. The second set of brackets).

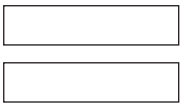
$3x - 4 \quad -2x - 3$	Remove brackets.
$x - 4 - 3$	Collect the "x" terms.
$x - 7$	Collect the numerical terms constants.

This can also be explained using algebra tiles.

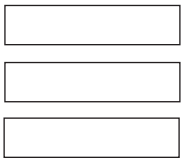
$(3x - 4) - (2x + 3)$



Beginning with the x term, remove



From



Left with

Now remove

From

Zero pairs will be needed in order to complete the subtraction.

Now the +3 can be removed, leaving behind –7.

The expression is $x - 7$.

Practice Questions

1. Combine the like terms in each expression.

   a. $2x^2 + 3x - 1 + x^2 - 4x - 2$

   = 3x² – x – 3

   b. $h^2 - 3h + 4h^2 + 5 - 7h + 2$

   = 5h² – 10h + 7

2. Add the polynomials.

   a. $(3x - 4) + (2x - 3)$

   = 5x – 7

   b. $(5a^2 - 3a + 2) + (-4a^2 + 2a - 3)$

   = a² – a – 1

3. What is the opposite of each expression?

   a. $3x - 7$

   –3x + 7

   b. $4g^2 - 4g + 3.5$

   –4g² + 4g – 3.5

   c. $v^2 + 8v - 1$

   –v² – 8v + 1

4. Subtract the polynomials.

   a. $(8h - 3) - (5h)$

   = 8h – 3 + –5h

   = 3h – 3

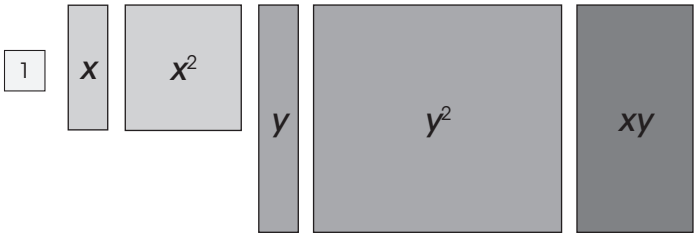
   b. $(6j^2 - 4j + 3) - (-2j^2 - 5)$

   = 6j² – 4j + 3 + 2j² + 5

   = 8j² – 4j + 8

Part 3: Multiplying and Dividing Monomials

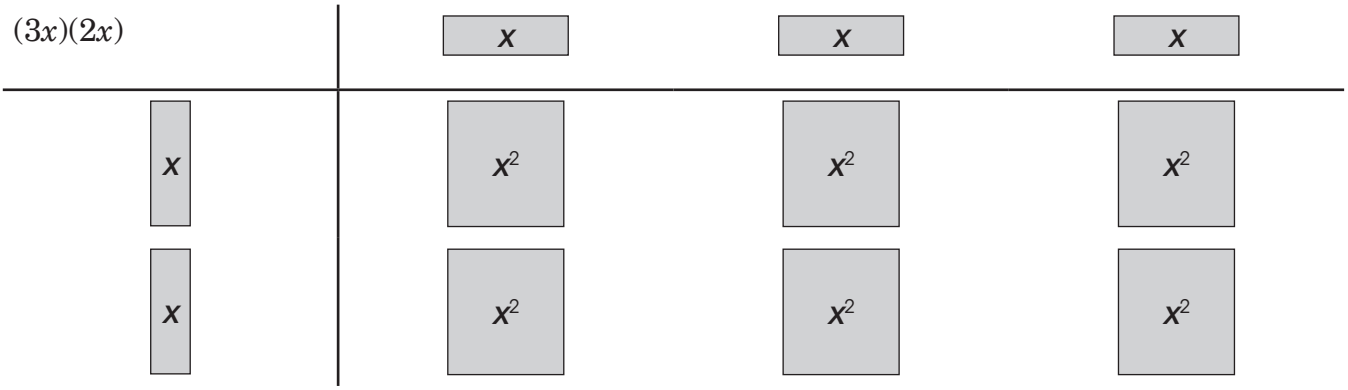
You can represent the multiplications and division of monomials and polynomials using models or algebraically. First, assign tiles to represent each expression.



Example 1 – Multiplying Monomials

To multiply monomials algebraically, multiply the numerical coefficients and use the exponent rules to multiply variables.

Find the product.



Each square has an area of $(x)(x) = x^2$. There are 6 positive x^2 tiles. So $(3x)(2x) = 6x^2$.

To complete the product algebraically:

1. Multiply the numerical coefficients.

2. Multiply the variables.
   $(3x)(2x)$
   $= (3)(2)(x)(x)$
   $= 6x^2$

Example 2 – Divide Monomials

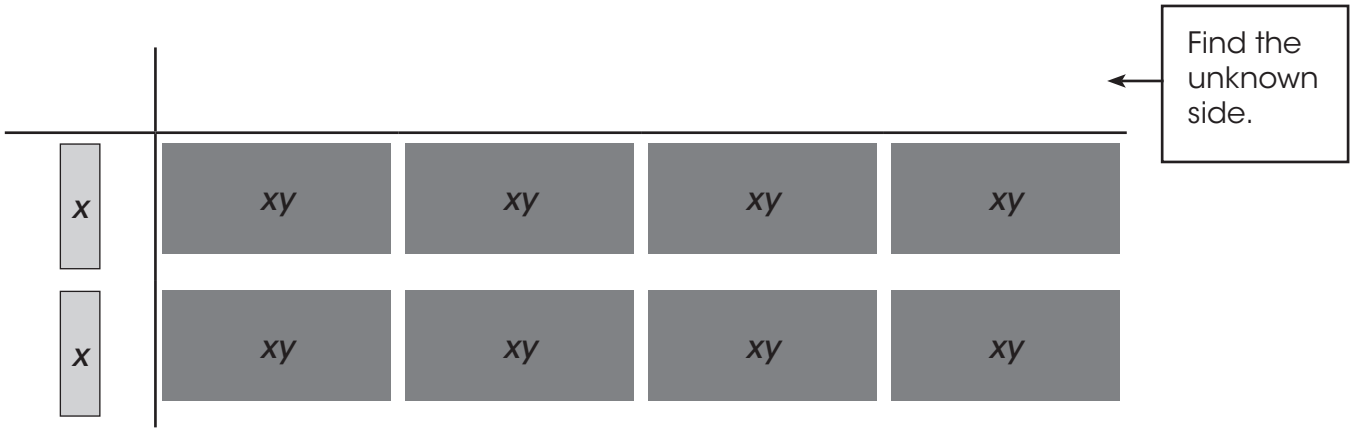
To divide monomials algebraically, divide the numerical coefficients and use the exponent rules to divide the variables.

Find the quotient: $\frac{8xy}{4x}$

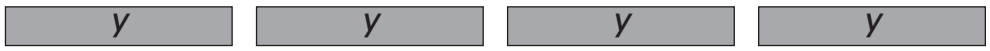
To complete the quotient algebraically, divide common factors in both the numerator and denominator.

$$\frac{8xy}{4x}$$
$$= \left(\cancel{8x}\, y\right)\left(\cancel{4x}\right)$$
$$= 2y$$

To model the quotient $\frac{8xy}{4x}$ with tiles, represent the numerator with $8xy$ tiles, then arrange the 8 tiles into a rectangle so that one of the sides is $2x$ tiles long.



The unknown side length of the rectangle is made up of $4y$ tiles.



Example 3 – Multiplying a Polynomial by a Monomial

When multiplying a polynomial, the Distributive Property is applied. This means the first term is multiplied by each of the following terms:

The Distributive Property means the first term is multiplied with each of the following terms:

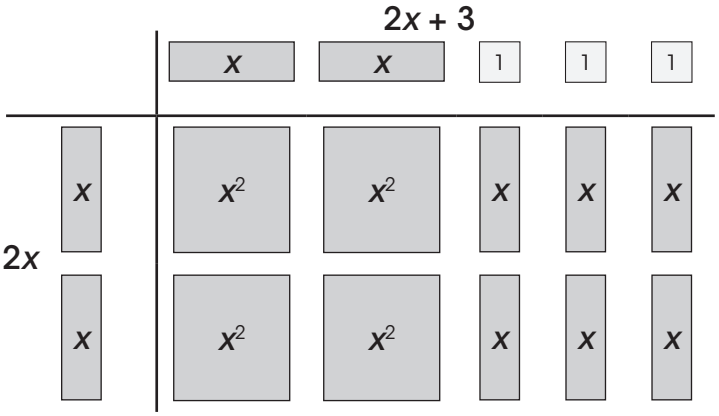
$(2x)(2x + 3)$

$(2x)\ (2x + 3)$

First, multiply $(2x)$ and $(2x) = 4x^2$
Then multiply $(2x)$ and $(3) = 6x$
$= 4x^2 + 6x$

To model it with tiles, it looks like this:

$(2x)\ (2x + 3)$



Therefore, **$(2x)\ (2x + 3) = 4x^2 + 6x$**

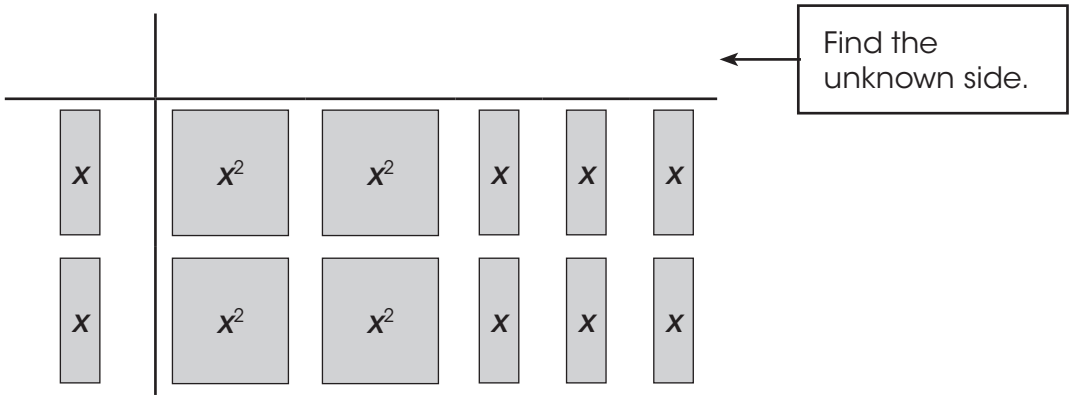
Example 4 – Dividing a Polynomial by a Monomial

When dividing a polynomial, divide the coefficients and apply the exponent laws to the variables.

$$\frac{4x^2 + 6x}{2x}$$
$$= \frac{4x^2}{2x} + \frac{6x}{2x}$$
$$= 2x + 3$$

To model it with tiles, it looks like this:

$$\frac{4x^2 + 6x}{2x}$$



The missing side is $2x + 3$.

Practice Questions

1. Solve.

   a. $(-q)(-4q)$

   = $4q^2$

   b. $(-3t)(4st)$

   = $-12st^2$

   c. $\frac{12x^2}{-3x}$

   = $-4x$

   d. $\frac{81rs}{-3rs}$

   = -27

2. Multiply.

   a. $(4j)(2j - 4)$

   = $8j^2 - 16j$

   b. $(3 - 9y)(y)$

   = $3y - 9y^2$

   c. $(8a - 7b - 4)\,(5a)$

   = $40a^2 - 35ab - 20a$

   d. $(-6x)(4 - 2.4x)$

   = $-24x + 14.4x^2$

3. Divide.

   a. $\frac{5x^2 - 10x}{5x}$

   = $x - 2$

   b. $\frac{12m^2 - 6.2m + 24}{2}$

   = $6m^2 - 3.1m + 12$

c. $\dfrac{-14a^2 - 7a + 0.5a}{0.5a}$

$= \dfrac{(\boldsymbol{-14a^2}) - (\boldsymbol{7a}) + (\boldsymbol{0.5a})}{(\boldsymbol{0.5a})\ (\boldsymbol{0.5a})\ (\boldsymbol{0.5a})}$

$\boldsymbol{-28a - 14 + 1}$

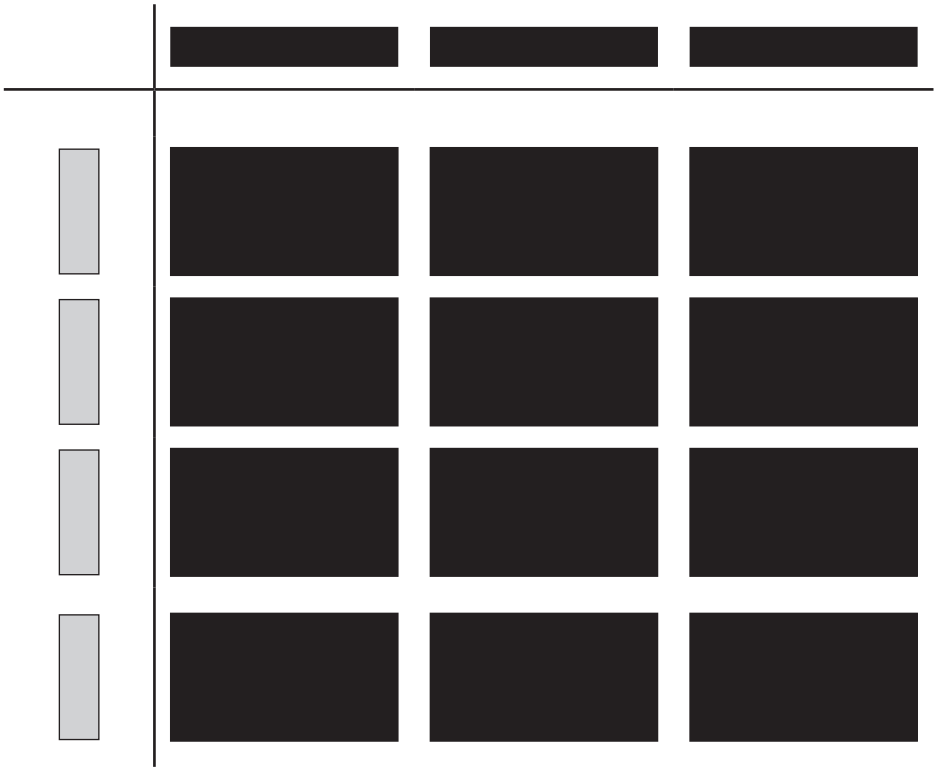
d. $\dfrac{-s^2 - 1.5st}{5s}$

$= \dfrac{(\boldsymbol{-s^2}) - (\boldsymbol{1.5st})}{(\boldsymbol{5s})\quad(\boldsymbol{5s})}$

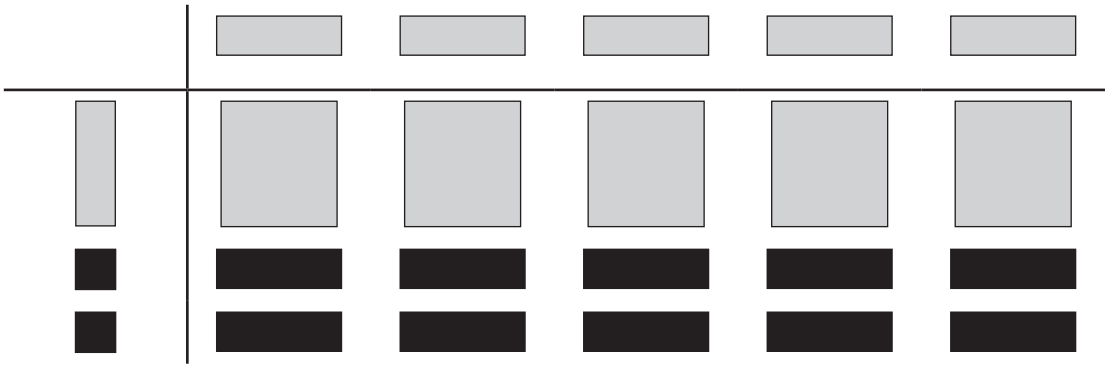
$= \boldsymbol{-0.2s - 0.3t}$

4. Model and solve the following expressions. Shade in the tiles with your pencil to indicate negative values.

$(4x)(-3y)$



$$\frac{5x^2 - 10x}{5x}$$



Lesson 9 Assignment

1. What is the degree of each polynomial?

   a. $6x^2$ b. $xy - 7x + 3$ c. $3 - x$

   second ***first*** ***first***

2. Complete the chart.

Expression	Number of Terms	Variable(s)
$5 - p + p - p^2$	***4***	***Polynomial***
$3p^3 - p$	***2***	***Binomial***
$-2a$	***1***	***Monomial***
$5x - 27x^2 + 2$	***3***	***Trinomial***

3. Combine the like terms.

   a. $4a + 3 + 9a + 1$

   = 13a + 4

   b. $2b^2 - 5b - 4b^2 + 8b$

   = –2b² + 3b

4. Simplify each expression.

   a. $(-p + 7) + (4p - 5)$

   = –p + 7 + 4p – 5
   = 3p + 2

   b. $(a^2 - a - 2) - (5 - 3a^2 + 6a)$

   = a² – a – 2 – 5 + 3a² – 6a
   = 4a² – 7a – 7

   c. $(5n - 1) - (4n + 7)$

   = 5n – 1 – 4n – 7
   = n – 8

   d. $(4x^2 - 5x - 1) + (2x + 5x^2 + 7)$

   = 4x² – 5x – 1 + 2x + 5x² + 7
   = 9x² – 3x + 6

5. Combine the like terms.

a. $(3n)(4n)$

$= 12n^2$

b. $(5xy)(5y)$

$= 25xy^2$

c. $\dfrac{7x^2}{-x}$

$= -7x$

d. $\dfrac{-18pn}{-3n}$

$= 6p$

6. Multiply.

a. $(5x)(2x + 1)$

$= 10x^2 + 5x$

b. $(3x)(x + 2y + 4)$

$= 3x^2 + 6xy + 12x$

c. $(6k + 2)(4.5k)$

$= 27k^2 + 9k$

d. $(-0.5m)(7 - 12m)$

$= -3.5m + 6m^2$

7. Divide.

a. $\dfrac{2x^2 - 8xy}{2x}$

$= \dfrac{(2x^2) - (8xy)}{(2x) \quad (2x)}$

$= x - 4y$

b. $\dfrac{12xy - 6x}{3}$

$= \dfrac{(-12xy) - (6x)}{(3) \quad (3)}$

$= -4xy - 2x$

c. $\dfrac{-2.7c^2 + 3.7c}{3c}$

$= \dfrac{(2.7c^2) + (3.7c)}{(3c) \quad (3c)}$

$= -0.9c + 1.2\overline{3}$

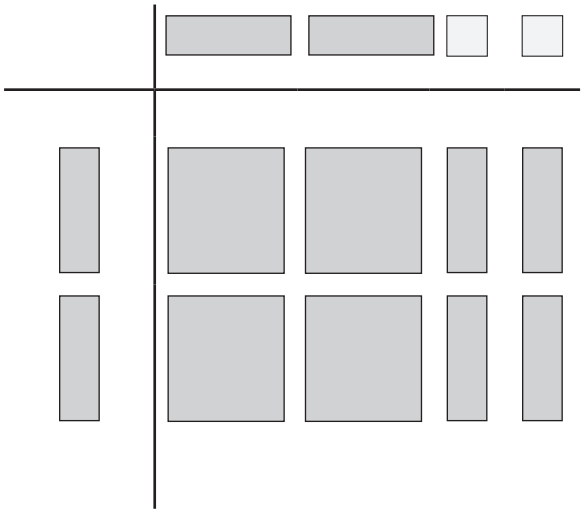
d. $\dfrac{12x^2 - 6x}{3x}$

$= \dfrac{(-12x^2) - (6x)}{(3x) \quad (3x)}$

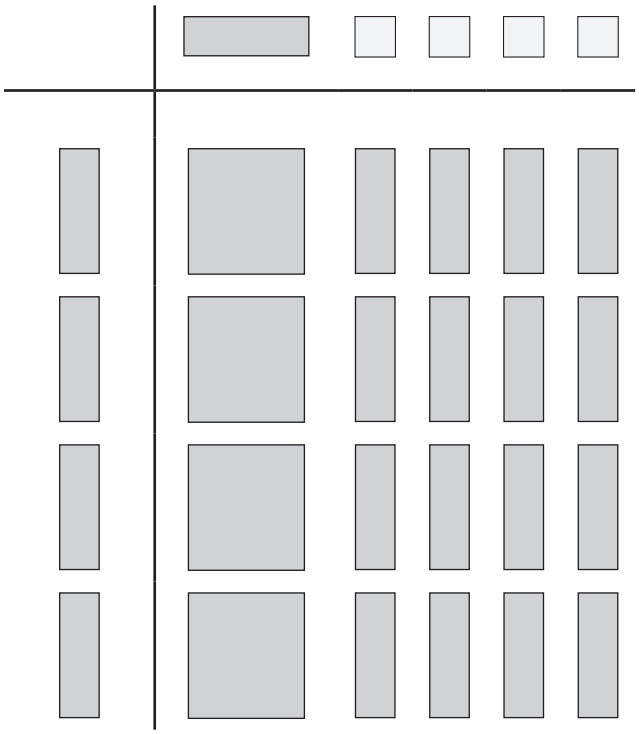
$= -4x - 2$

8. Model and solve the following expressions. Shade in the tiles with your pencil to indicate negative values.

$(2x)(x+2)$



$$\frac{4x^2+16x}{4x}$$




LEARNING
ANYTIME
ANYWHERE