Important Concepts . . .

Preview Review

Mathematics Grade 9 TEACHER KEY

W2 - Review

Important Concepts of Grade 9 Mathematics

Materials Required

Pencil
Paper
Calculator
Algebra Tiles

No Textbook Required

This is a stand-alone course.

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Preview/Review W2 - Review
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Preview/Review Concepts for Grade Nine Mathematics

Teacher Key

W2 – Review

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Materials required:

- Paper, Pencil, Calculator, Algebra Tiles

Part 1: Graphing Linear Relations

Many patterns, both pictorial and written can be represented using a table of values. This table of values can also be expressed as a linear equation.

Example 1

x	y
–1	–1
0	2
1	5
2	8
3	11

The equation that represents this table of value s is $y = 3x + 2$

Linear relations can be checked by substituting in the value that was solved.

Example 2

Using the above example,

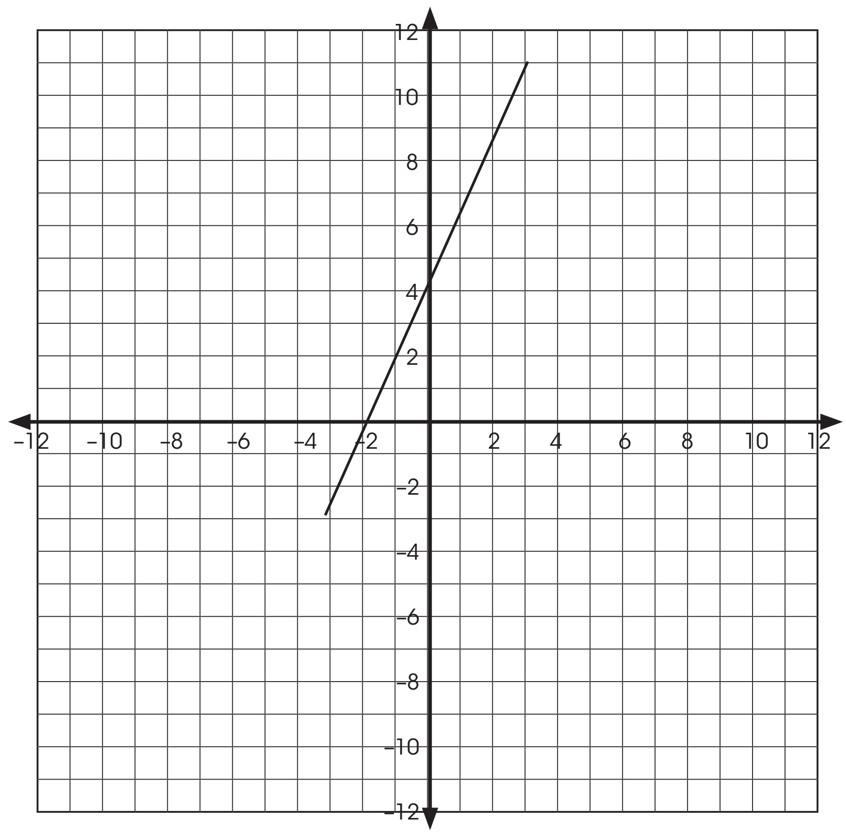
$y = 3x + 2$ Choose an "x" value from the table of values, for example "3".
$y = 3(3) + 2$ Substitute the value of $x = 3$.
$y = 9 + 2$ Check back to the table of value to see if when $x = 5$, does $y = 11$?

Yes. The solution is verified.

Example 3

Linear relations can also be graphed to find unknown values. Given the linear relation $y = 3x + 2$, substitute in values for x to determine the values for y. Fill in the table of values. Then, plot the points on a graph.

$y = 3x + 2$	
x	y
–1	***–1***
0	***2***
1	***5***
2	***8***
3	***11***



Extending the line can also determine unknown values. For example, the line would extend to point (–2 , –4). This indicates that if $x = –2$, then y would equal –4.

Practice Questions

1. Discover the pattern in the following table of values. Complete the table.

a.

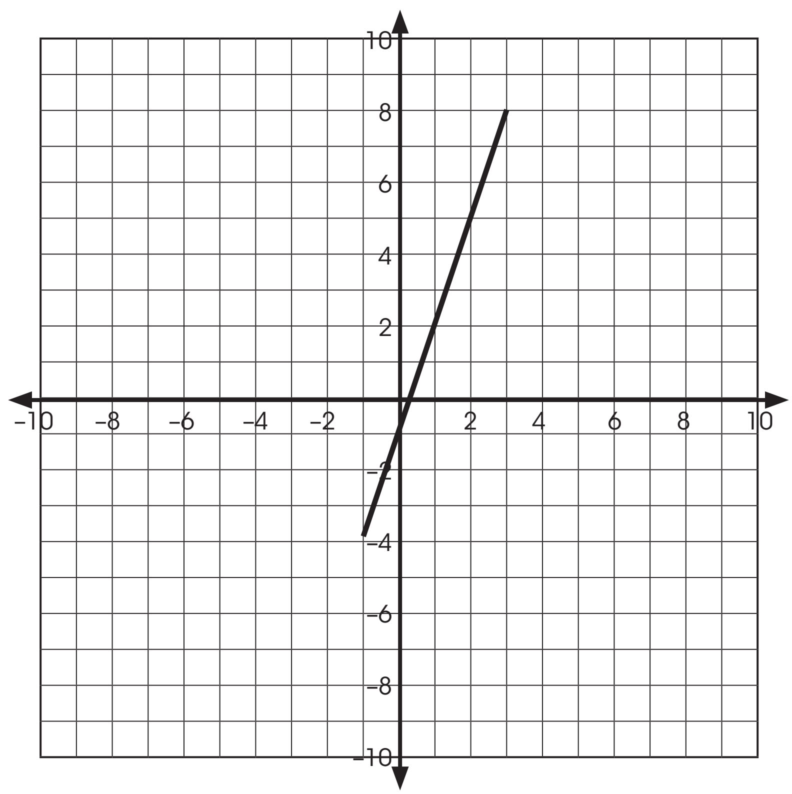
x	y
2	6
3	10
4	14
5	***18***
6	***22***
7	***26***

b. What is the linear relation for the table above? ***4x – 2***

2. Create a table of values and graph the following equation.

$y = 3x - 1$

x	y
–1	***–4***
0	***–1***
1	***2***
2	***5***
3	***8***



Part 2: Solving Linear Relations

An equation is a number sentence containing a variable and an = sign .

Some examples of equations are:

$ax = b$ $\qquad \dfrac{x}{a} = b$ $\qquad \dfrac{a}{x} = b$

When solving these types of equations, only one step is required.

To verify an answer, simply substitute the solution in the equation, then check to make sure the right side = the left side.

Example 1

$$\frac{x}{3} = 27$$
$$\frac{x}{3}(3) = 27(3)$$
$$x = 81$$

Verification

$$\frac{x}{3} = 27$$
$$\frac{(81)}{3} = 27$$
$$27 = 27$$

Left side = Right side

Example 2

There are also equations that require two steps to solve for an unknown variable. These equations can be in the form:

$ax + b = c$ or $\frac{x}{a} + b = c$

To solve these linear equations isolate the variables. Two-step equations will require two processes to solve for the variable. Using inverse operations will solve for a variable.

- The inverse of addition is subtraction.
- The inverse of division is multiplication.

Solve

$$\begin{aligned} 2x + 2 &= 10 \\ 2x + 2 - 2 &= 10 - 2 \\ 2x &= 8 \\ \frac{2x}{2} &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

To isolate the variable in a two step equation, use the reverse order of operations. Add or subtract first, then multiply or divide.

Verification

$$\begin{aligned} 2x + 2 &= 10 \\ 2(4) + 2 &= 10 \\ 10 &= 10 \end{aligned}$$

Right side = Left side

Example 3

To isolate the variable in equations of the form $a(x + b) = c$, consider the following steps.

$$\begin{aligned} 4(f+5) &= 36 \\ 4f+20 &= 36 \\ 4f+20-20 &= 36-20 \\ 4f &= 16 \\ f &= 4 \end{aligned}$$

Verification

$$\begin{aligned} 4(f+5) &= 36 \\ 4(4+5) &= 36 \\ 4(9) &= 36 \\ 36 &= 36 \end{aligned}$$

Right side = Left side

Example 4

The equations shown in this format contain variables on both sides of the equations. To solve for the variable, it must be isolated:

$10d = 25(d - 30)$

$$\begin{aligned} 10d &= 25(d-30) \\ 10d &= 25d-750 \\ 10d-25d &= 25d-25d-750 \\ -15d &= -750 \\ \frac{-15d}{-15} &= \frac{-750}{-15} \\ d &= 50 \end{aligned}$$

Use the distributive property.

Verification

$$\begin{aligned} 10d &= 25(d-30) \\ 10(50) &= 25(50-30) \\ 500 &= 25(20) \\ 500 &= 500 \end{aligned}$$

Left side = Right side

Example 5

The equations shown in this format contain fractions. To solve for the variable, first remove all fractions by multiplying each term by the LCD (lowest common denominator).

$$\begin{aligned} 4n-\frac{2}{10}&=\frac{3}{5} \\ (10)4n-(10)\frac{2}{10}&=(10)\frac{3}{5} \\ 40n-2&=6 \\ 40n&=8 \\ n&=5 \end{aligned}$$

The lowest common denominator of 10 and 5 is 10. So, multiply each term by 10.

Practice Questions

1. Solve for each of the following. Verify your answers for b and d.

a. $2v = -\dfrac{5}{6}$

$$\begin{aligned}(2v)(6) &= -5\\ 12v &= -5\\ v &= -\frac{5}{12}\end{aligned}$$

b. $\dfrac{3}{5} = \dfrac{a}{4}$

$$\begin{aligned}\left(\frac{3}{5}\right)(4) &= \frac{a}{4}(4)\\ \frac{12}{5} &= a\\ a &= 2\frac{2}{5}\end{aligned}$$

c. $3x = 21$

$$\begin{aligned}\frac{3x}{3} &= \frac{21}{3}\\ x &= 7\end{aligned}$$

Verification

$$\begin{aligned}3x &= 21\\ 3(7) &= 21\\ 21 &= 21\end{aligned}$$

$$\textit{Left side} = \textit{Right side}$$

b. $\dfrac{14}{x} = 7$

$$\begin{aligned}14 &= 7x\\ \frac{14}{7} &= x\\ 2 &= x\end{aligned}$$

Verification

$$\begin{aligned}\frac{14}{x} &= 7\\ \frac{14}{2} &= 7\\ 7 &= 7\end{aligned}$$

$$\textit{Left side} = \textit{Right side}$$

2. Solve for each of the following.

a. $2.05 = 0.9x - 6.5$

$$\begin{aligned} 2.05 + 6.5 &= 0.9x - 6.5 + 6.5 \\ 8.55 &= 0.9x \\ x &= 9.5 \end{aligned}$$

b. $12.4g + 34.3 = 9.5 - 3.1g$

$$\begin{aligned} 12.4g + 3.1g + 34.3 &= 9.5 - 3.1g + 3.1g \\ 15.5g + 34.3 - 34.3 &= 9.5 - 34.3 \\ 15.5g &= -24.8 \\ g &= -1.6 \end{aligned}$$

c. $-\frac{4}{5}(q+1) = -3$

$$\begin{aligned} -\frac{4}{5}q - \frac{4}{5} &= -3 \\ (5)-\frac{4}{5}q + (5)-\frac{4}{5} &= (5)-3 \\ -4q - 4 &= -15 \\ -4q - 4 + 4 &= -15 + 4 \\ -4q &= -11 \\ q &= \frac{11}{4} = 2\frac{2}{3} \end{aligned}$$

d. $10.1 = 0.9 + \frac{h}{-2}$

$$\begin{aligned} (2)10.1 &= (2)0.9 + (2)\frac{h}{-2} \\ 20.2 &= 1.8 - h \\ 20.2 - 1.8 &= 1.8 - 1.8 - h \\ 18.4 &= -h \\ h &= -18.4 \end{aligned}$$

3. Solve the following two step equations.

a. $\frac{3}{4} - \frac{r}{3} = \frac{7}{8}$

$$\begin{aligned}(24)\frac{3}{4} - (24)\frac{r}{3} &= (24)\frac{7}{8}\\ 18 - 8r &= 21\\ -8r &= 3\\ r &= -\frac{3}{8}\end{aligned}$$

b. $2z = 4(2z - 3)$

$$\begin{aligned}2z &= 8z - 12\\ -6z &= -12\\ z &= 2\end{aligned}$$

c. $\frac{1}{2} + \frac{b}{6} = \frac{5}{12}$

$$\begin{aligned}(12)\frac{1}{2} + (12)\frac{b}{6} &= (12)\frac{5}{12}\\ 6 + 2b &= 5\\ 2b &= -1\\ b &= -\frac{1}{2}\end{aligned}$$

d. $4\,(y - 2) = 3(y + 8)$

$$\begin{aligned}4y - 8 &= 3y + 24\\ 4y - 3y - 8 &= 3y - 3y + 24\\ y - 8 &= 24\\ y - 8 + 8 &= 24 + 8\\ y &= 32\end{aligned}$$

e. $0.25m = 0.75\,(m + 40)$

$$\begin{aligned}0.25m &= 0.75m + 30\\ 0.25m - 0.75m &= 0.75m - 0.75m + 30\\ -0.5m &= 30\\ \frac{-0.5}{-0.5m} &= \frac{30}{-0.5m}\\ m &= -60\end{aligned}$$

f. $5.60p + 18.75 = 46.26 - 2.26p$

$$\begin{aligned}5.60p + 18.75 - 18.75 &= 46.26 - 18.75 - 2.26p\\ 5.60p &= 27.51 - 2.26p\\ 5.60p + 2.26p &= 27.51 + 2.26p - 2.26p\\ 7.86p &= 27.51\\ p &= 3.5\end{aligned}$$

Part 3: Inequalities

An inequality compares a linear expression that may or may not be equal. The symbols used for inequality are:

$>$ means ‘greater than’
$<$ means ‘less than’
$\geq$ means ‘greater than or equal to’
$\leq$ means ‘less than or equal to’

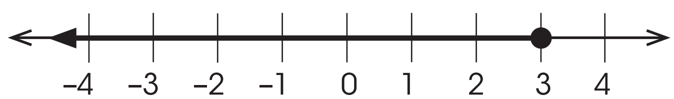
For example, $x \leq -5$ means that the unknown value, x, is less than or equal to –5.

Example 1

An inequality can be represented graphically.

$y > -5$	

The above line segment beginning at –5 is not coloured in completely; this shows $>$.

$x \leq 3$	

The above line segment beginning at 0 is completely coloured in. This represents $\leq$.

Example 2

Solving an inequality is very similar to solving an equation. The purpose is to isolate the variable to solve for a solution.

$$\begin{aligned} x-5 &\le 10 \\ x-5+5 &\le 10+5 \\ x &\le 15 \end{aligned}$$

When each side of the inequality is multiplied or divided by a negative number, the inequality sign must be reversed in order for it to be true.

$$\begin{aligned} -8x &\ge 24 \\ \frac{-8x}{-8} &\le \frac{24}{-8} \\ x &\le -3 \end{aligned}$$

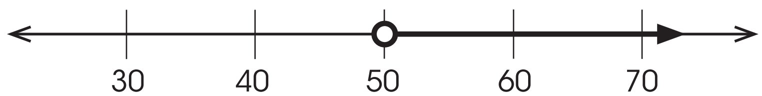
To solve multi-step inequalities, work step by step to isolate the variable, remembering to reverse the inequality sign only when multiplying or dividing by a negative number.

$$\begin{aligned} 5x-19 &< 36 \\ 5x-19+19 &< 36+19 \\ 5x &< 45 \\ \frac{5x}{5} &< \frac{45}{5} \\ x &< 9 \end{aligned}$$

Practice Questions

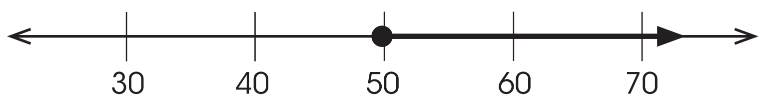
1. Write the inequality that represents the following number lines.

a.



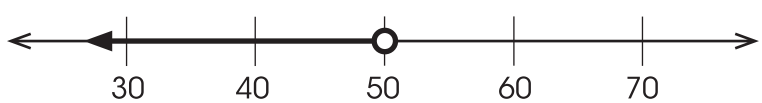
***x* > 50**

b.



***x* ≥ 50**

c.



***x* < 50**

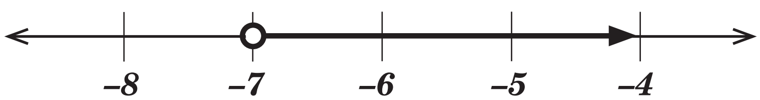
2. Solve the following inequalities. Graph the solution for b and c.

a. $x - 7 > 22$

$$\boldsymbol{x - 7 + 7 > 22 + 7}$$
$$\boldsymbol{x > 29}$$

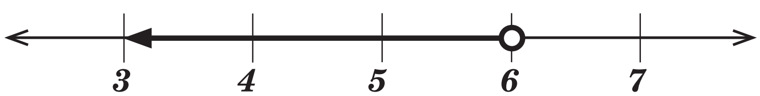
b. $4 < x + 11$

$$\boldsymbol{4 - 11 < x + 11 - 11}$$
$$\boldsymbol{-7 < x}$$



c. $7x < 2x + 30$

$$\boldsymbol{7x - 2x < 2x - 2x + 30}$$
$$\boldsymbol{5x < 30}$$
$$\boldsymbol{x < 6}$$



d. $-12x + 10 > 19 - 4x$

$$\boldsymbol{-12x + 10 - 10 > 19 - 10 - 4x}$$
$$\boldsymbol{-12x > 9 - 4x}$$
$$\boldsymbol{-12x + 4x > 9 - 4x + 4x}$$
$$\boldsymbol{-8x > 9}$$
$$\boldsymbol{\frac{-8x}{-8} < \frac{9}{-8}}$$
$$\boldsymbol{x < -\frac{9}{8} = -1\frac{1}{8}}$$

Part 4: Polynomials

A polynomial is an algebraic expression that is made up of terms containing variables, coefficients, exponents and constants. These terms are connected together by addition and subtraction operations.

Polynomials can be named by the number of terms in the expression.

- Monomial – has one term.
- Binomial – has two terms.
- Trinomial – has three terms.

$7x^2 + 5x - 8$ This expression has three terms so it is a trinomial. $7x^2$ is the term with the highest degree; it has a degree is 2. Therefore, the degree of this polynomial is 2.

Example 1

To add and subtract polynomials, it is important to collect like terms. Like terms differ only by their numerical coefficients.

Examples of like terms are:

$-9x$ and $14x$ $16z^2$ and $-4z^2$ $8ab$ and $-ab$

Addition and subtraction equations can be solved algebraically or by using algebra tiles. To solve algebraically:

$(3x + 2) + (2x - 3)$

$= 3x + 2 + 2x - 3$ Remove the brackets.
$= 5x + 2 - 3$ Collect like terms with common variables.
$= 5x - 1$ Collect the numerical terms (constants).

Example 2

When finding the difference in polynomials, determine the opposite polynomials. Use the opposite strategy to subtract polynomials.

$(4x - 3) - (3x + 2)$

$(4x - 3) + (-3x - 2)$	Add the opposite.
$4x - 3 - 3x - 2$	Remove brackets.
$x - 3 - 2$	Collect like terms with common variables.
$x = -5$	Collect the numerical terms (constants).

Example 3

To multiply polynomials, multiply the numerical coefficients and use the exponent rules to multiply the variables.

$(4x)(5x)$

$(4x)(5x)$ Multiply the coefficients.

$(4x)(5x)$ Multiply the variables.

$= 20x^2$

Example 4

To divide polynomials algebraically, divide the numerical coefficients and use the exponent rules to divide the variables.

$$\frac{12xy}{4x}$$

$\frac{12xy}{4x}$ Divide the numerical coefficients.

$\frac{12xy}{4x}$ Apply exponent laws to variables.

$= 3y$

Example 5

To multiply polynomials algebraically, expand the expression using the distributive property.

$(4x)(-2x + 5)$

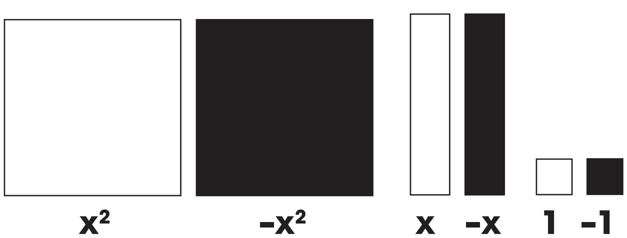
$(4x)(-2x + 5)$ First, multiply $(4x)$ and $(-2x) = -8x^2$.

$(4x)(-2x + 5)$ Then multiply $(4x)$ and $(5) = 20x$.

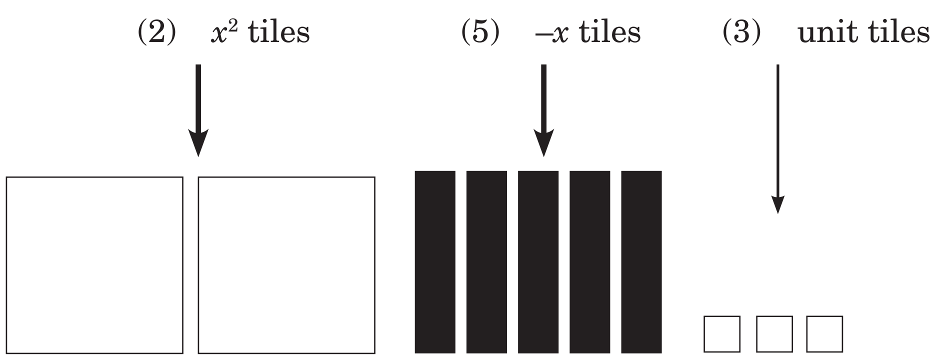
$= -8x^2 + 12x$

Example 6

Polynomials can be modelled with algebra tiles.



To model a polynomial like $2x^2 - 5x + 3$, simply select the corresponding tiles:



Practice Questions

1. Use the polynomial $10x^2 - 2x - 4$ to answer the following questions.

   a. What type of polynomial is this? ***trinomial***

   b. What is the degree? ***second***

   c. List the variable(s). ***x***

   d. List the exponent(s). ***2***

   e. List the coefficient(s). ***10, –2***

   f. List the constant(s). ***–4***

2. Solve the following by adding or subtracting the polynomials.

   a. $(3x^2 - 8x + 6) + (9x^2 + 4x - 1)$

   $= 12x^2 - 4x + 5$

   b. $(8 - 6v) + (3 - 8v)$

   $= 11 - 14v$

   c. $(9m - 4) - (3 - 8m)$

   $= 9m - 4 + -3 + 8m$
   $= 17m - 7$

   d. $(2xy - 4x + y) - (7xy - 3x - y)$

   $= 2xy - 4x + y + -7xy + 3x + y$
   $= -5xy - x + 2y$

3. Find the product.

a. $(-32x)(-2y)$

= 64xy

b. $(5a)(-4a - 2)$

= –20a² – 10a

c. $(-4x)(-3x + 2)$

= 12x² – 8x

d. $(5x)(3x - 4)$

= 15x² – 20x

4. Find the quotient.

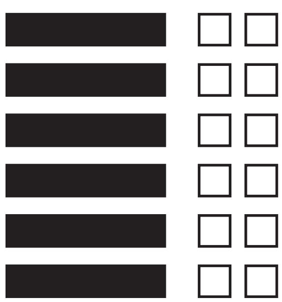
a. $\frac{14n^2 - 2n}{2n}$

$= \boldsymbol{7n - 1}$

b. $\frac{5x^2 - 3x}{-3}$

$= \boldsymbol{\frac{5x^2}{-3} + x}$

___***B***___ 5. Which of the following quotients is represented in the tiles below?



A. $\frac{6x - 12}{6}$

B. $\frac{-6x + 12}{6}$

C. $\frac{-6x - 12}{6}$

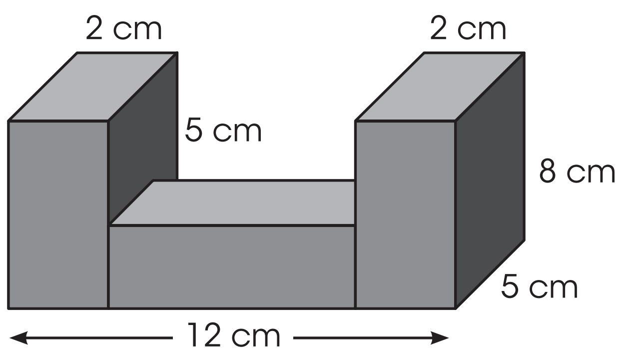
Part 5: Surface Area of Composite Shapes

To determine the surface area of a 3D composite object, it is important to determine how the composite object is constructed.

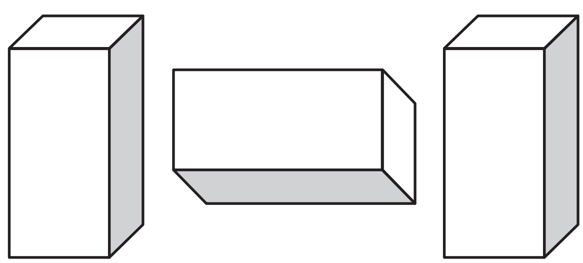
- Using symmetry, determine similar faces.
- Determine the area of each face.
- Add these areas together.
- Consider how the composite shape is constructed and remove any overlapping areas.

Example 1

Consider the following 3D composite object.

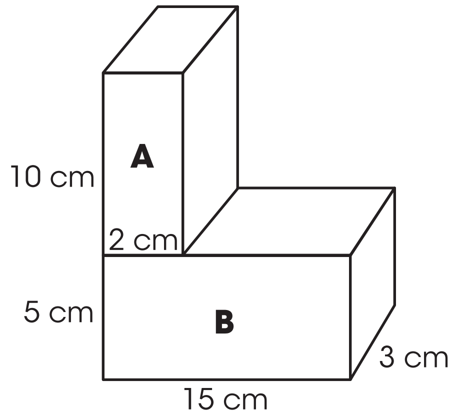


This object is constructed of the following pieces:



Example 2

What is the surface area of the following shape?



***SA* Piece A**

$SA = 2(lh) + 2(lw) + 2(hw)$
$SA = 2(10 \times 2) + 2(3 \times 2) + 2(10 \times 3)$
$SA = 40 + 12 + 60$
$SA = 112 \text{ cm}^2$

***SA* Piece B**

$SA = 2(lh) + 2(lw) + 2(hw)$
$SA = 2(15 \times 5) + 2(3 \times 5) + 2(15 \times 3)$
$SA = 150 + 30 + 90$
$SA = 270 \text{ cm}^2$

Overlap on Piece A (Same Overlap on Piece B)

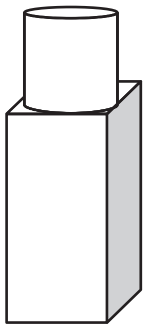
Area = lw
$A = 3 \times 2$
$A = 6 \text{ cm}^2$

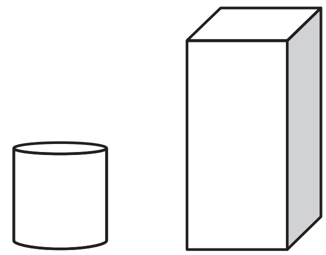
Total $SA = A + B -$ Overlaps
$SA = 112 + 270 - 6 - 6$
$SA = 370 \text{ cm}^2$

Practice Questions

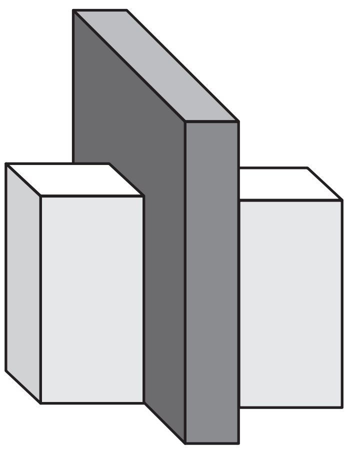
1. Draw the objects that represent each composite.

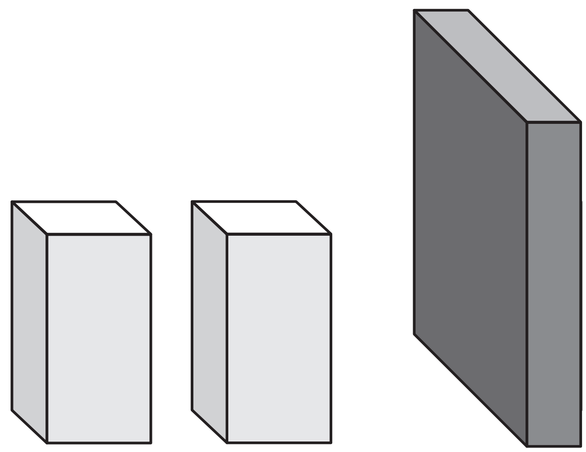
a.



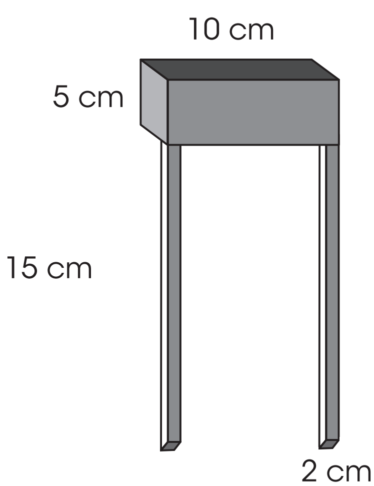


b.





2. Calculate the surface area of the following composite object.



SA Top Shape

$SA = 2(lh) + 2(lw) + 2(hw)$
$SA = 2(10 \times 5) + 2(10 \times 5) + 2(5 \times 5)$
$SA = 100 + 100 + 50$
$SA = 250 \text{ cm}^2$

SA Base Shape $\times 2$
$SA = 2(lh) + 2(lw) + 2(hw)$
$SA = 2(2 \times 15) + 2(2 \times 2) + 2(15 \times 2)$
$SA = 60 + 8 + 60$
$SA = 128 \text{ cm}^2 \times 2 = 256 \text{ cm}^2$

Overlap (on top shape and on column) $\times$ ***2 columns***
Area $= lw$
$A = 2 \times 2$
$A = 4 \text{ cm}^2 \times 2 = 8 \times 2 \text{ cm}^2 = 16 \text{ cm}^2$

Total SA = A + B – Overlaps
$SA = 250 + 256 - 16$
$SA = 490 \text{ cm}^2$

LEARNING ANYTIME ANYWHERE