Important Concepts . . .

Preview Review

Mathematics Grade 9 TEACHER KEY

W3 - Lesson 15: Probability

Important Concepts of Grade 9 Mathematics

Materials Required

Paper
Pencil
Calculator

No Textbook Required

This is a stand-alone course.

Mathematics Grade 9
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Preview/Review W3 - Lesson 15
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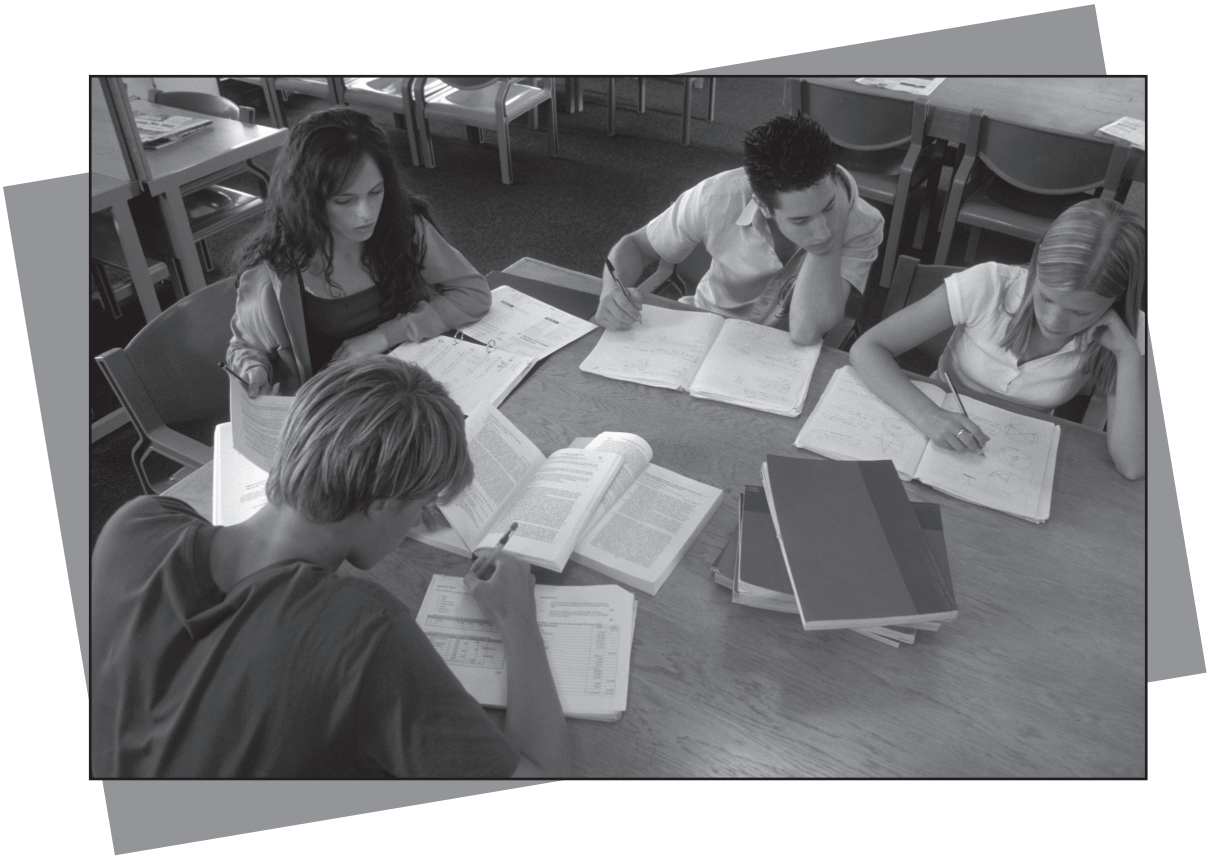
Preview/Review Concepts for Grade Nine Mathematics

Teacher Key



W3 – Lesson 15:

Probability

OBJECTIVES

By the end of this lesson, you will be able to:

- Explain, using examples, how decisions may be based on a combination of theoretical probability, experimental probability and subjective judgement.

GLOSSARY

Probability: The chance of an event occurring.

Theoretical Probability: What is EXPECTED will happen in a probability experiment.

Experimental Probability: What ACTUALLY happens in a probability experiment.

Outcome: A possible result of an event.

Possible Outcome: All of the possibilities of an event.

Favourable Outcome: What outcome is desired to happen.

Event: A set of one or more outcomes in a probability experiment.
Ex. flipping a coin consists of 2 outcomes: heads or tails.

Independent Events: When the occurrence of one event does not depend on the occurrence of another event.

W3 – Lesson 15: Probability

Materials required:

- Paper, Pencil, and Calculator

Part 1: Independent Events

$$\textbf{Theoretical Probability} = \frac{\textbf{Favourable outcomes}}{\textbf{Possible outcoms}}$$

Number of ways to get desired result.

Total number of possibilities.

- Write the fraction in reduced form
- Can change the fraction to a decimal or percent

Example 1

Picture a 6-sided die. Record each of the following as a fraction, decimal and percent.

What is the theoretical probability of rolling a 6?

There is 1 favourable (desired) outcome.

There are 6 possible outcomes.

$$\text{Probability of } (6) = \frac{1}{6}$$

Two events are said to be independent when the occurrence of one event does not affect the occurrence of another event. An example of this is the flipping of two coins. Flipping one coin does not affect how the other coin will land.

What are all the possible outcomes of flipping two coins? Complete the Carroll diagram.

Coin 1	*Heads*	*Tails*
Coin 2		
Heads	Heads/Heads	Heads/Tails
Tails	Tails/Heads	Tails/Tails

There are four possible outcomes. The probability of each outcome is:

Heads/Heads: $\frac{1}{4}$

Heads/Tails: $\frac{2}{4}$ or $\frac{1}{2}$

Tails/ Tails: $\frac{1}{4}$

Therefore, to determine the probability of two independent events :

Probability (*A* and *B*) = Probability(*A*) × Probabilty(*B*), where A is the first event and B is the second event.

So in the example of the probability of landing Heads/Heads,

$$P(A) = \frac{1}{2} \text{ and } P(B) = \frac{1}{2}$$

$$\frac{1}{2} \square \frac{1}{2} = \frac{1}{4}$$

$$P(A \square B) = \frac{1}{4}$$

The probability of flipping two coins and landing Heads/Heads is $\frac{1}{4}$.

Example 2

What is the probability of rolling a 5 on a 6-sided die and tossing a tails on a coin at the same time?

Let A = rolling a 5 on a 6-sided die
Let B = tossing a tails on a coin

Applying the formula shows:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$
$$P(5 \text{ and tails}) = P(5) \cdot P(\text{Tails})$$
$$= \frac{1}{6} \cdot \frac{1}{2}$$
$$= \frac{1}{12}$$

The probability of rolling a 5 on a 6-sided die **and** tossing a tails on a coin is $\frac{1}{12}$.

Practice Questions

1. Suppose two dice are rolled, with each die containing 6-sides. What is the probability of:

   a. Rolling two 6's?

   $$\frac{1}{36}$$

   b. Rolling an even number?

   $$\frac{3}{6} \times \frac{3}{6} = \frac{9}{36} = \frac{1}{4}$$

   a. Rolling a sum of 7?

   1 and 6, 6 and 1
   3 and 4, 4 and 3
   2 and 5, 5 and 2
   $$\frac{6 \text{ favourable}}{36 \text{ total}} = \frac{1}{6}$$

   b. Rolling an even number, then an odd ?

   $$\frac{3}{6} \times \frac{3}{6} = \frac{9}{36} = \frac{1}{4}$$

2. Suppose a coin is flipped and a 6-sided die is rolled.

   a. Create a chart to show all of the possible outcomes.

Side on the Die	Side on the Coin
1	*Head*
1	*Tail*
2	*Head*
2	*Tail*
3	*Head*
3	*Tail*
4	*Head*
4	*Tail*
5	*Head*
5	*Tail*
6	*Head*
6	*Tail*

   b. What is the theoretical probability of each outcome? Express the answer as a fraction, percent and decimal.

Each outcome has a $\frac{1}{12}$ ***chance or 0.0833...or 8.33%***

Part 2: Using Probability to Make Decisions

Probability can help people make decisions. Probability allows us to predict the possible outcomes of certain events occurring and allows us to predict when certain outcomes could be more favourable to occur.

Compare the theoretical and experimental probability:

$$\frac{\textbf{Theoretical Probability}}{\textit{(what } \textbf{\textit{should}} \textit{ happen)}} = \frac{\textbf{Favourable outcomes}}{\textbf{Possible outcomes}}$$

$$\frac{\textbf{Experimental Probability}}{\textit{(what } \textbf{\textit{does}} \textit{ happen)}} = \frac{\textbf{Number of times event occurred}}{\textbf{Number of times conducted}}$$

Probability can be:

- converted to reduced fractions
- made into equivalent fractions
- converted to decimals, percents, ratios

Example: What is the theoretical probability that of rolling a 4 on a 5-sided die?

$P(4) = \frac{1}{5}$

Now conduct the experiment.

Roll a 5-sided die 12 times. Complete the chart below by filling in the number you rolled into each box.

3	4	5	1	3	2
5	2	4	5	3	4

How many times did you roll the die? 12

What was the experimental probability that the die rolled a 4? $P(4) = \frac{2}{12}$ or $\frac{1}{6}$

How do the theoretical and experimental probabilities compare?

The theoretical probability is $\frac{1}{5}$. The experimental probability is $\frac{1}{6}$.

The theoretical probability was greater.

Example 1

Each chart shows the results of a probability experiment. Complete the questions for each chart.

Coin Toss	
Heads	**Tails**
	

1. What is the theoretical probability of landing heads? $\frac{1}{2}$

2. What is the theoretical probability of landing tails? $\frac{1}{2}$

3. How many times was the coin tossed? ***15***

4. What is the experimental probability of the coin landing heads? $\frac{6}{15} = \frac{2}{5}$

5. What is the experimental probability of the coin landing tails? $\frac{9}{15} = \frac{3}{5}$

6. How do the experimental probability and theoretical probability compare?

   The theoretical probability is that the coin has a 50% chance of landing either heads or tails. The experimental probability is that the coin has 40% chance of landing heads, and a 60% chance of landing tails.

Practice Questions

Kai, Keenan and Andrew created a game with the following materials: 2 small squares of blue construction paper, 4 small squares of green construction paper, 6 small squares of red construction paper and a paper bag

Each player was assigned a colour.

Kai: **Blue**
Keenan: ***Green***
Andrew: *Red*

They placed all the markers in the bag. They would each reach into the bag and pull out one marker. The player who had the most markers drawn would win the round and shade their square in the round. All markers were replaced before the next round. The player who receives the most points wins the game.

Here are the results of their game.

Round	#1: Kai	#2: Keenan	#3: Andrew
1	**Blue**	***Green***	***Green***
2	*Red*	*Red*	***Green***
3	*Red*	**Blue**	*Red*
4	***Green***	***Green***	***Green***
5	**Blue**	*Red*	**Blue**
6	***Green***	*Red*	*Red*
7	*Red*	*Red*	***Green***
8	*Red*	***Green***	*Red*
9	*Red*	*Red*	*Red*
10	***Green***	*Red*	*Red*

1. Who do you think will win? Why?

   Andrew because he has more markers in the bag.

2. Who do you predict to be last? Why?

   Kai because he has the fewest number of markers in the bag.

3. What is the theoretical probability of drawing a green marker? A blue marker? A red marker?

Green : $\frac{4}{12} = \frac{1}{3}$ ***Blue :*** $\frac{2}{12} = \frac{1}{6}$ ***Red :*** $\frac{6}{12} = \frac{1}{2}$

4. How many draws were made in total?

30

5. What is the experimental probability of drawing a green marker? A blue marker? A red marker?

Green : $\frac{10}{30}$ ***Blue :*** $\frac{4}{30}$ ***Red :*** $\frac{16}{30}$

6. Compare the experimental probability and the theoretical probability of the green marker. How do they compare? Express your answers as a percent.

Experimental $= \frac{1}{3} = 33\frac{1}{3}\%$, ***Theoretical*** $= \frac{1}{3} = 33\frac{1}{3}\%$. ***They are the same.***

7. Compare the experimental probability and the theoretical probability of the blue marker. How do they compare? Express your answers as a percent.

Experimental $= \frac{4}{30} = 13\frac{1}{3}\%$, ***Theoretical*** $= \frac{1}{6} = 16\frac{2}{3}\%$.

The theoretical probability is greater.

8. Compare the experimental probability and the theoretical probability of the red marker. How do they compare? Express your answers as a percent.

Experimental $= \frac{16}{30} = 53\frac{1}{3}\%$, ***Theoretical*** $= \frac{1}{2} = 50\%$.

The experimental probability is greater.

9. Was this a fair game? Explain.

No because Andrew has a greater chance of winning.

Part 3: Probability in Society

Probability occurs in society as events, surveys and natural occurrences. Using experimental and theoretical probability helps makes decisions based on probability.

Example 1

There are 400 students at River Brook Jr. High School. There are 100 students in grade 6, 100 students in grade 7, 100 students in grade 8 and 100 students in grade 9. Each student in each grade was asked what leisure activity they prefer the most. The results of the survey are presented in the chart below.

Grade	Skiing	Biking	Watching TV	Computer Activities	Total
6	13	19	40	19	100
7	27	12	57	4	100
8	11	24	59	11	100
9	14	9	44	37	100

a. What is the probability that a member at any grade level will choose skiing as an activity? Use this to predict how many members in theory would choose skiing. Does this match the actual outcome?

Since there are 4 choices, the probability of choosing skiing is $\frac{1}{4}$ or 25%
If there are 400 students, then 400 × 0.25 (25%) = 100
The total number of students who chose skiing was 65.
This number is lower than the predicted outcome.

b. What assumptions do you make regarding each outcome?

The assumptions that were made were that every activity had an equal chance of being selected and all students had an equal interest level in the surveyed activities.

c. Based on the survey, predict the number of students that will choose swimming.

$$\frac{13+27+11+14}{400} = \frac{65}{400} = 16.25\%$$

Practice Questions

1. Movie Muster is an on-line DVD rental company. They have 5 genres of movies: comedy, drama, horror, action and science fiction. Kaden is renting a movie.

   a. What is the theoretical probability that Kaden choose a drama?

   $$\frac{1}{5} = 20\%$$

   b. What assumption did you make when calculating this probability?

   The assumption is that each genre has an equal chance of being selected by Kaden.

2. The following chart represents the DVD rentals from the last 50 customers of Movie Muster.

Genre of Movie	Rentals
Comedy	15
Drama	10
Horror	12
Action	11
Sci-Fi	2

   a. Predict the probability that a consumer will choose an action movie.

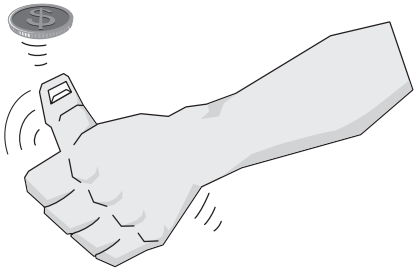
   $$\frac{11}{50} = 22\%$$

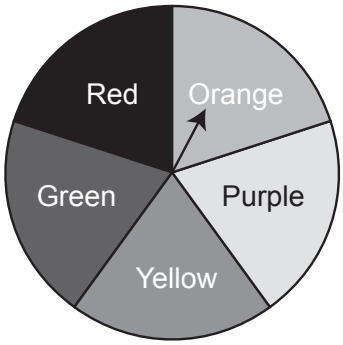
   b. About how many customers out of 2000 will choose a drama movie? Explain.

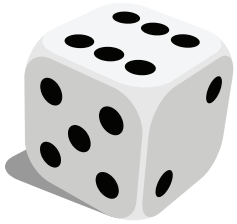
   $$\frac{10}{50} = \frac{1}{5} = 20\%$$
   $$0.2 \times 2000 = 400$$

Lesson 15 Assignment

1. Calculate all of the probabilities if you had the following materials:







a. Spinning a green and rolling a 5.

$$\frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$$

b. Rolling an odd number and tossing tails.

$$\frac{3}{6} \times \frac{1}{2} = \frac{3}{12} = \frac{1}{4}$$

c. Not spinning yellow and rolling an even number.

$$\frac{4}{5} \times \frac{3}{6} = \frac{12}{30} = \frac{2}{5}$$

d. Spinning orange or red, rolling an even number and tossing heads.

$$\frac{2}{5} \times \frac{3}{6} \times \frac{1}{2} = \frac{6}{60} = \frac{1}{10}$$

2. Ashley has 15 loose socks in her drawer. 5 of her socks are white, 6 of her socks are black and 4 of her socks are blue.

   a. What is the theoretical probability that Ashley will pull out a white sock?

   $$\frac{5}{15} = \frac{1}{3}$$

   b. What is the theoretical probability that Ashley will pull out a blue sock?

   $$\frac{4}{15}$$

3. Emilio placed 5 white linking cubes, 6 black linking cubes, and 4 blue linking cubes in a bag. He replaced the cube after each draw. He did this for a total of 20 times. This chart records his results.

White	Black	Blue	Black	Blue	Black	White	Black	White	Black
Black	Blue	Black	Blue	White	Black	Blue	White	Black	Black

   a. What is the experimental probability that Emilio will pull out a white cube?

   $$\frac{5}{20} = \frac{1}{4}$$

   b. What is the experimental probability that Emilio will pull out a blue cube?

   $$\frac{5}{20} = \frac{1}{4}$$

   c. If Emilio repeated his experiment 60 times, how many times would be the theoretical possibility the cube would be white?

   $\frac{5}{20}$ ***is the theoretical probability for a white cube, therefore,*** $\frac{15}{60}$ ***would be the probability of 60 experiments.***

4. The town of Coalhurst is having an election for Mayor. Coalhurst has 4000 citizens. Pradeesh Ramaj, Jack Frost and Wendy Lu are the three candidates. A poll was conducted by surveying 40 people. It was found that of these 40 people surveyed, 53% would votes for Pradeesh, 23% would votes for Jack and 24% would votes for Wendy.

   a. How many people polled said they would vote for Wendy?

   40% of 24 = 9.6

   About 9 or 10 people said they would vote for Wendy.

   b. What is the theoretical probability that a voter will choose Pradeesh? What is the experimental probability that a voter will choose Pradeesh?

   There are 3 candidates, therefore the theoretical probability is $\frac{1}{3}$ or $33\frac{1}{3}$%. 53% is the experimental probability, since 53% of the voters said they would vote for Pradeesh.

   c. What assumptions are made when calculating the theoretical probability?

   The assumption is that all candidates have an equal chance of being selected; all voters are equally likely to choose any candidate.

   d. Compare the experimental and theoretical probabilities of Jack winning the election.

   Experimental : $\frac{9}{40} = 23\%$ Theory : $\frac{1}{3} = 33\%$

   The theoretical probability of Jack winning the election is greater than the experimental probability.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