

Important Concepts . . .

Preview Review



Mathematics Grade 9

W3 - Lesson 11: Properties of Circles

Important Concepts of Grade 9 Mathematics	Materials Required
W1 - Lesson 1 Powers W1 - Lesson 2 Exponents W1 - Lesson 3 Rational Numbers W1 - Lesson 4 Order of Operations W1 - Lesson 5 Square Roots of Rational Numbers W1 - Review W1 - Quiz	Paper Pencil Calculator
W2 - Lesson 6 Graphing Linear Relations W2 - Lesson 7 Solving Linear Relations W2 - Lesson 8 Linear Inequalities W2 - Lesson 9 Polynomials W2 - Lesson 10 Surface Area of 3D Objects W2 - Review W2 - Quiz	No Textbook Required This is a stand-alone course.
W3 - Lesson 11 Properties of Circles W3 - Lesson 12 Polygons and Scale Diagrams W3 - Lesson 13 Rotational Symmetry W3 - Lesson 14 Representing Data W3 - Lesson 15 Probability W3 - Review W3 - Quiz	

Mathematics Grade 9

Version 6

Preview/Review W3 - Lesson 11

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Preview/Review Concepts for Grade Nine Mathematics



W3 - Lesson 11:
Properties of Circles

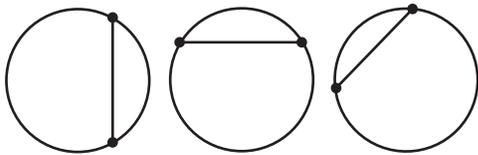
OBJECTIVES

By the end of this lesson, you will be able to:

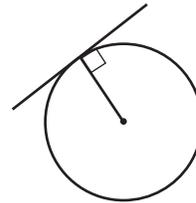
- Solve a given problem involving application of one or more of the circle properties.
- Determine the measure of a given angle inscribed in a semicircle, using the circle properties.
- Explain the relationship among the centre of a circle, a chord and the perpendicular bisector of the chord.

GLOSSARY

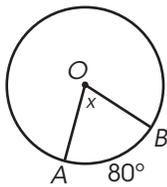
Chord: Is a segment that joins two points of the circle.



Tangent: A tangent is a line intersecting only one point on the circle; it is perpendicular to the radius.



Central Angle: A central angle is an angle formed by two intersecting radii such that its vertex is at the center of the circle.



W2 – Lesson 11: Properties of Circles

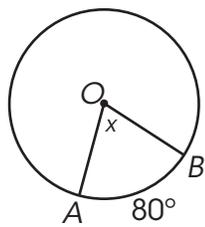
Materials required:

- Paper, Pencil, and Calculator

Part 1: Central Angle Property

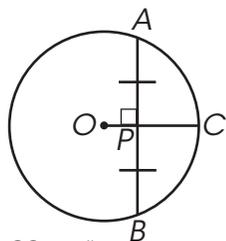
Properties related to angles in circles can be used to solve problems. To solve problems, properties of a circle need to be defined.

The central angle is an angle formed by two radii of a circle.



The central angle is x . This angle is formed by the radii lines AO and BO .

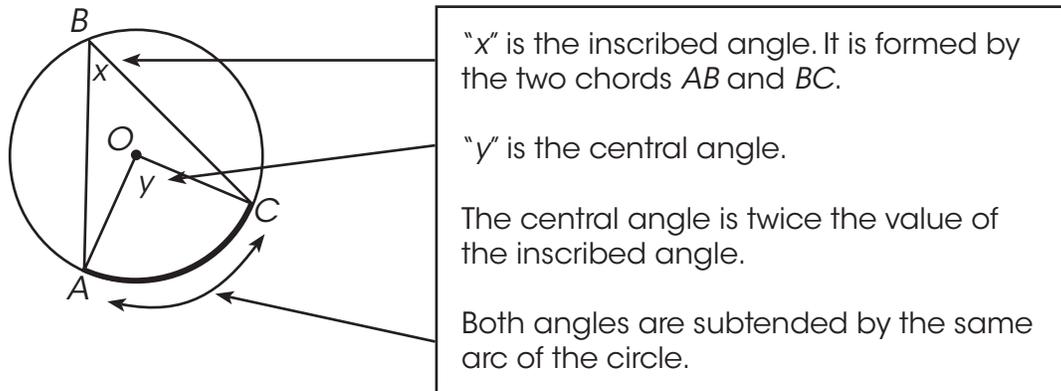
A chord is a line segment with both end points of the line segment falling somewhere on the circle.



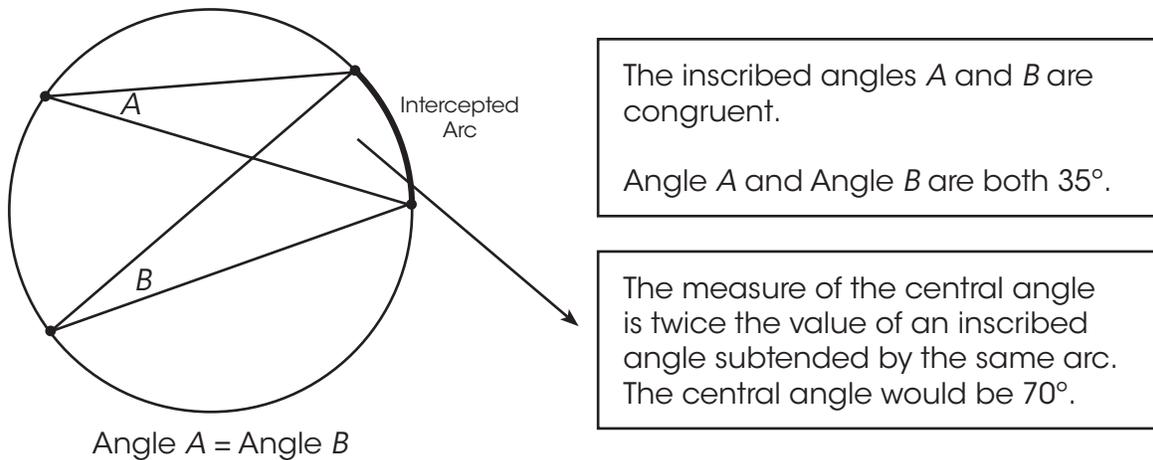
$OC = \text{radius}$
 $AB = \text{chord}$

The chord AB had end points that fall on the circle.

An inscribed angle is an angle formed by two chords that share a common end point, and are subtended by the same arc of the circle.



Any inscribed angles on a circle subtended by the same arc are congruent.



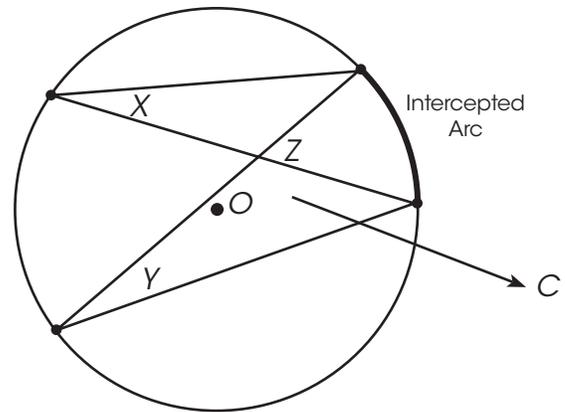
Example 1

“O” is the centre of the circle.
 $\angle X$ is 35° .

What is the measure of $\angle Y$?
 Because $\angle X$ and $\angle Y$ share the same subtend arc,
 $\angle X = \angle Y$. So $\angle Y = 35^\circ$.

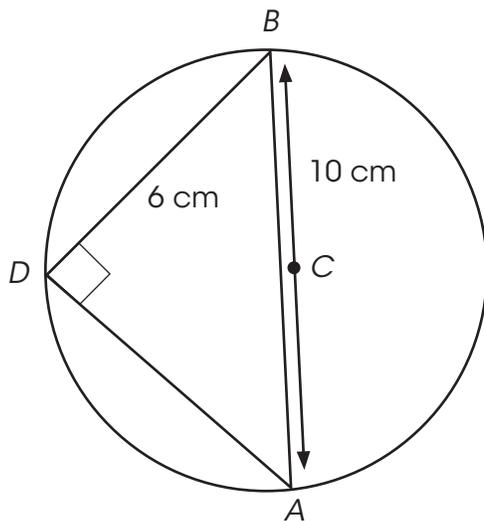
What is the measure of $\angle Z$?

The measure of the central angle is equal to twice the measure of the inscribe angle.
 So the measure of $\angle Z$ is 70° .



Angle A = Angle B

Example 2



Point C is the center of the circle. The diameter of this circle is 10 cm. The diameter is represented by the line AB. There are two chords in this circle.

- a. What is the length is the chord AD?

Because triangle ADB is a right angled triangle, to find the length of chord AD, use the Pythagorean Theorem ($a^2 + b^2 = c^2$) to find the length of AD.

$$AD^2 + BD^2 = AB^2$$

$$AD^2 + 6^2 = 10^2$$

$$AD^2 + 36 = 100$$

$$AD^2 = 100 - 36$$

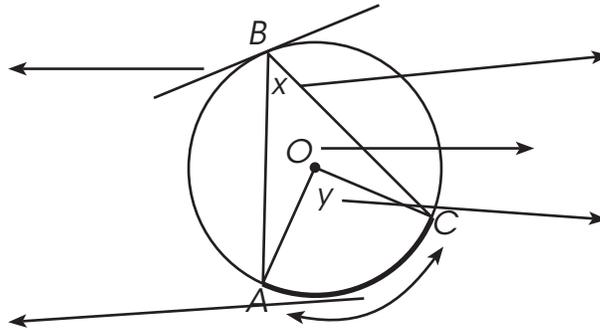
$$AD^2 = 64$$

$$AD^2 = \sqrt{64}$$

$$AD^2 = 8 \text{ cm}$$

Practice Questions

1. Label the following on the diagram below: *centre of the circle*, *tangent line*, *arc*, *inscribed angle*, and *central angle*.



2. Determine the following:
 - a. The inscribed angle on a given circle is 45° . What is the measurement of the central angle?

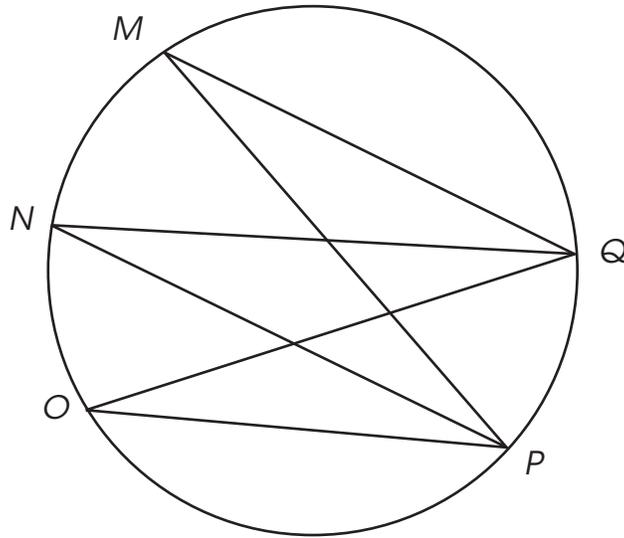
 - b. The inscribed angle on a given circle is 25° . What is the measurement of the central angle?

 - c. The central angle on a given circle is 144° . What is the measurement of the inscribed angle?

 - d. The central angle on a given circle is 120° . What is the measurement of the inscribed angle?

 - e. Two inscribed angles share the same arc, therefore they are congruent. If the value of one angle is 73° , what is the value of the other inscribed angle?

3. Using inscribed angle properties, determine which 3 angles are congruent.
 Circle the three congruent angles below.



$\angle M$

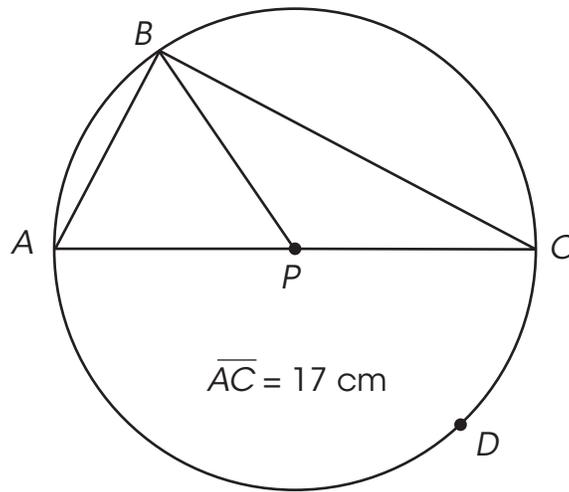
$\angle N$

$\angle O$

$\angle P$

$\angle Q$

4. Point P is the center of the circle. The diameter of this circle is 17 cm. The diameter is represented by the line AC . There are two chords in this circle. Chord BC is 15 cm long.



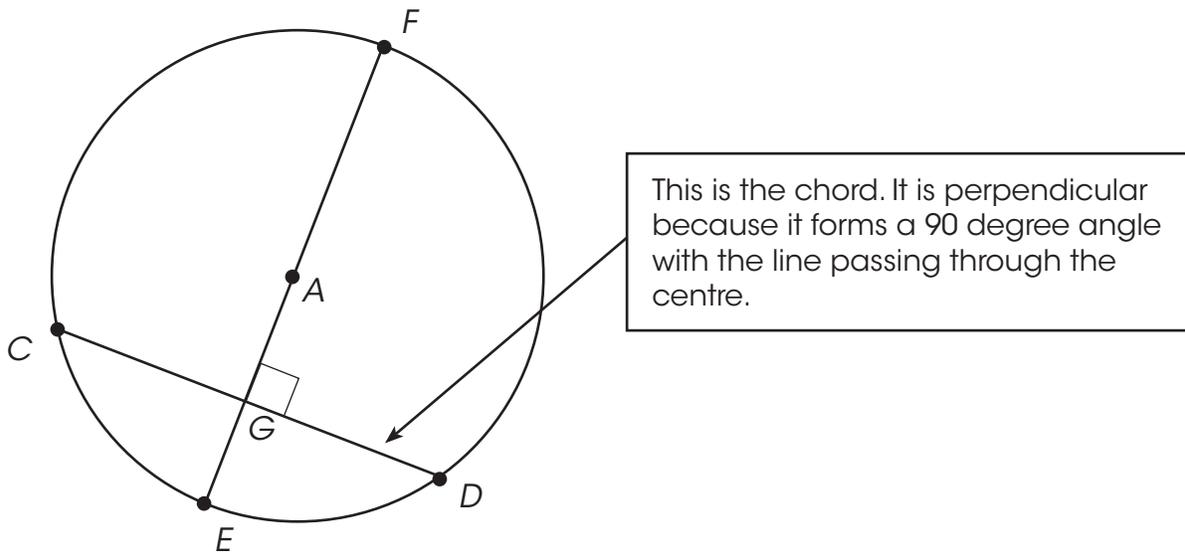
- a. What is the measure of $\angle ABC$?

- b. What is the length of the chord AB ?

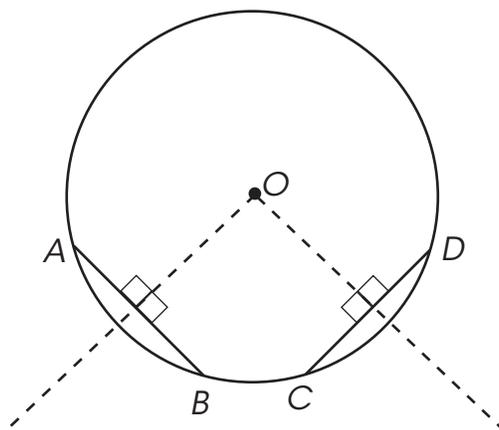
Part 2: Chord Properties

If a line passes through the centre of a circle and intersects a chord at right angles, then the line bisects the chord.

The perpendicular bisector of a chord passes through the center of the circle.



When a bisector of a chord passes through the centre of the circle, the bisector is perpendicular to the chord.

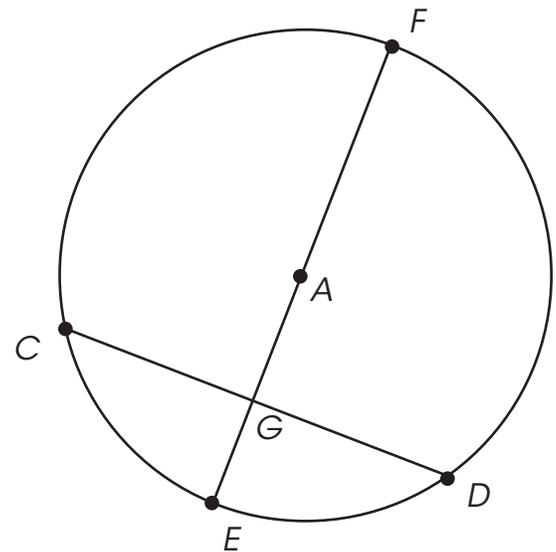


Example 1

The radius AE bisects chord CD . AG measures 4 mm. Chord CD measures 14 mm. What is the radius of the circle?

Drawing a radius from AD . This will form a right angled triangle. Now apply Pythagorean Theorem to solve.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 4^2 + 7^2 &= AD^2 \\
 16 + 49 &= AD^2 \\
 65 &= AD^2 \\
 \sqrt{65} &= AD \\
 8.1 &= AD
 \end{aligned}$$



The radius of the circle is 8.1 mm.

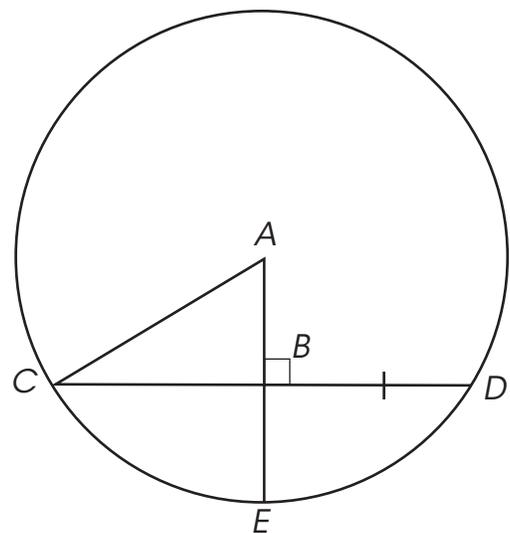
Example 2

AE bisects chord CD . The diameter of the circle is 30 cm long. Chord CD measures 24 cm. What is the length of AB ?

Since triangle ABC is a right angled triangle, it is possible to solve for the length of AB using the Pythagorean Theorem.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 12^2 + AB^2 &= 15^2 \\
 144 + AB^2 &= 225 \\
 AB^2 &= 81 \\
 AB &= \sqrt{81} \\
 AB &= 9 \text{ cm}
 \end{aligned}$$

\overline{AB} is 9 cm long.



Practice Questions

1. The center of the circle is O .

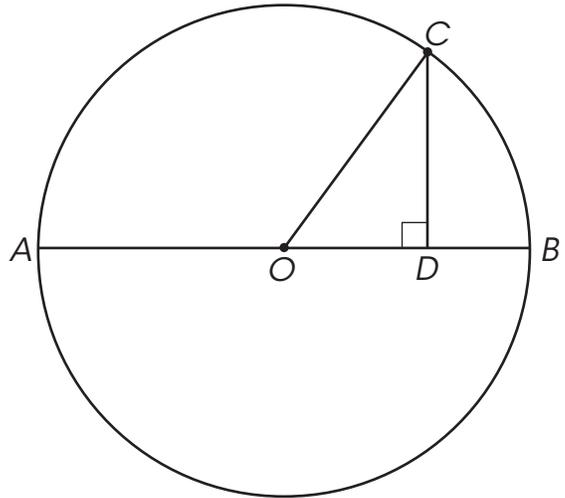
$AO = 5 \text{ cm}$

$DB = 2 \text{ cm}$

- a. What is the length of OC ?

- b. What is the length of OD ?

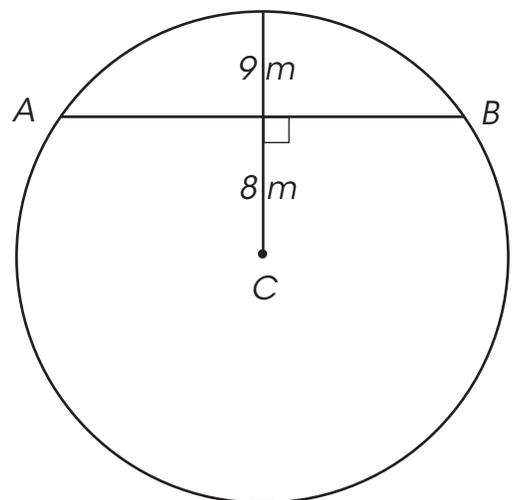
- c. What is the length of CD ?



2. Look at the following diagram. The chord is 80 m long. The radius is 17 m long

- a. What is the distance between points C and A ?

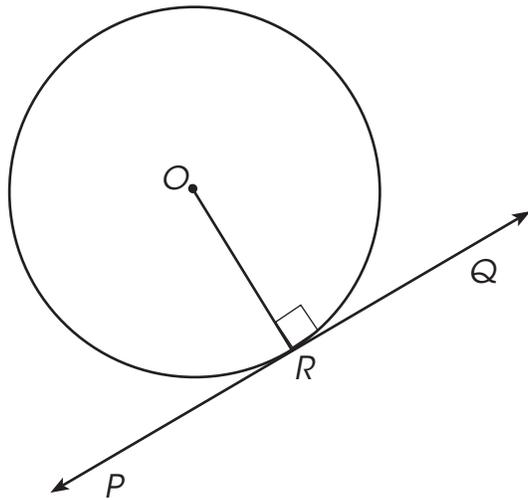
- b. What is the distance between points C and B ?



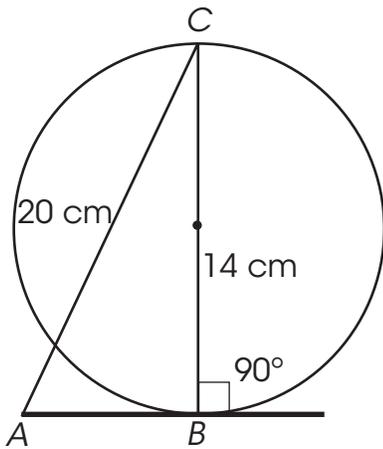
Part 3: Tangent Properties

A tangent is a line that touches a circle at exactly one point. The point where the line touches the circle is called the point of tangency.

The properties of tangents to a circle can be used to solve problems.



Tangent to a Circle: A tangent to a circle is perpendicular to the radius at the point of tangency.
R is the point of tangency.



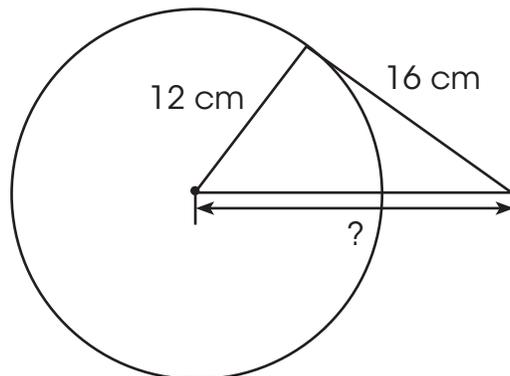
Tangent Chord Relationship: A chord that is drawn perpendicular to a tangent of a circle, at the point of tangency, will contain the centre of the circle. This is the diameter.

Example 1

What is the measurement of the unknown length?

Since a tangent to a circle is perpendicular to the radius at the point of tangency, this creates a right triangle. Use the Pythagorean Theorem to find the unknown side.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 12^2 + 16^2 &= c^2 \\
 144 + 256 &= c^2 \\
 400 &= c^2 \\
 \sqrt{400} &= c \\
 c &= 20
 \end{aligned}$$



The unknown side is 20 cm.

Example 2

The smaller triangle is an isosceles triangle because it contains two radii of the circle and the radii are of equal length. Therefore, the other angle is also 63°.

Drawing a tangent line creates a 90° angle with the centre of the circle. If one angle is 63°, then $90^\circ - 63^\circ = 27^\circ$.

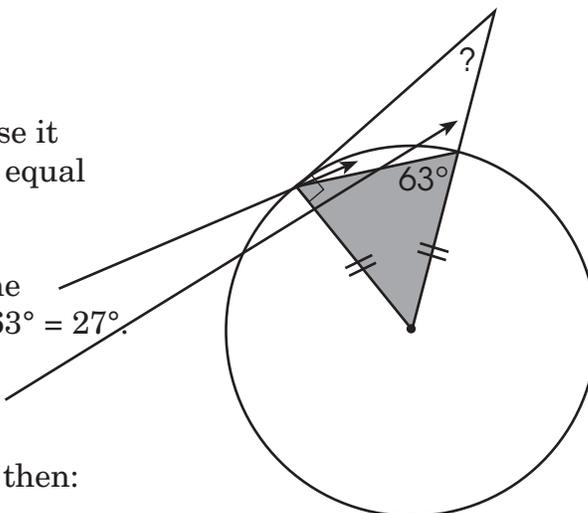
A straight line is 180°, therefore $180^\circ - 63^\circ = 117^\circ$

Since the sum of the angles in any triangle is 180, then:

$$27^\circ + 117^\circ + \text{unknown angle} = 180$$

$$180^\circ - 117^\circ - 27^\circ = 36^\circ$$

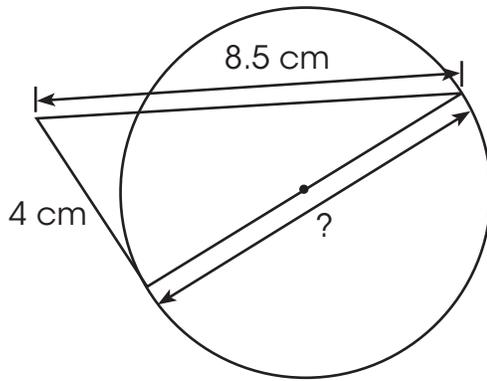
The unknown angle is 36°.



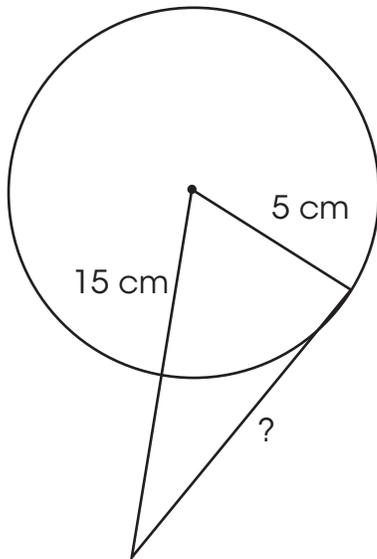
Practice Questions

1. Find the missing length.

a.

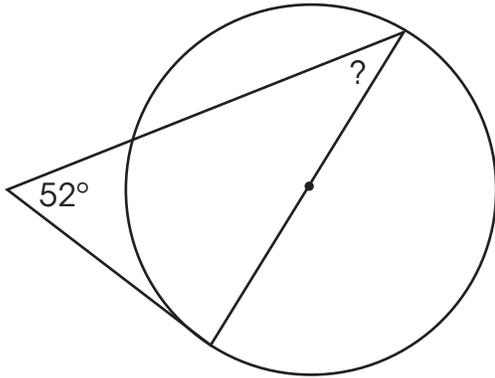


b.

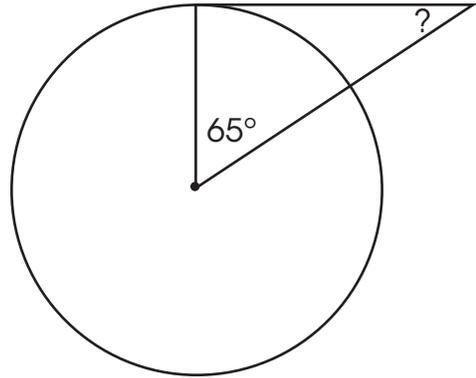


2. Find the missing length.

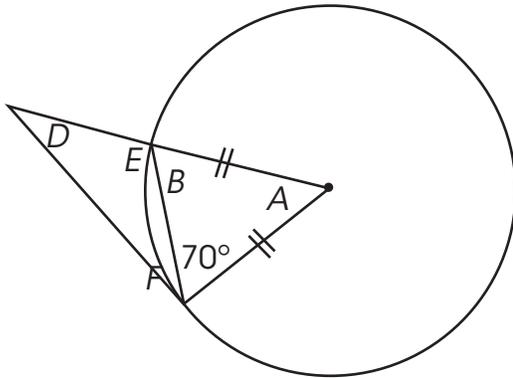
a.



b.



c.

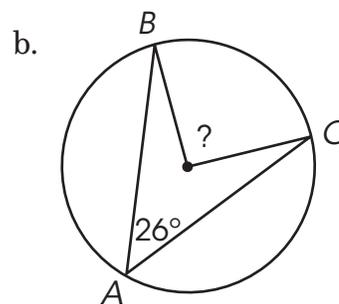
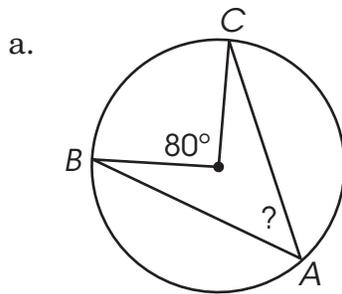


Lesson 11 Assignment

1. Label the following parts of the circle:

	X refers to the
	Y refers to the
	O refers to the
	Z refers to the
	The arrow refers to the

2. Find the measure of the angle indicated.



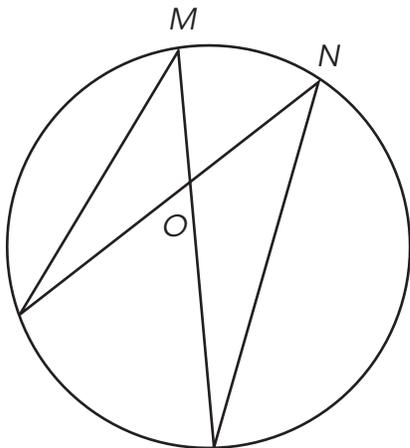
3. Answer the following:

a. The inscribed angle on a given circle is 33° . What is the measurement of the central angle?

b. The central angle on a given circle is 128° . What is the measurement of the inscribed angle?

c. Two inscribed angles share the same arc, therefore they are congruent. If the value of one angle is 61° , what is the value of the other inscribed angle?

4. If the value of O is 96° , what is the value of M ? What is the value of N ?



5. The center of the circle is O .

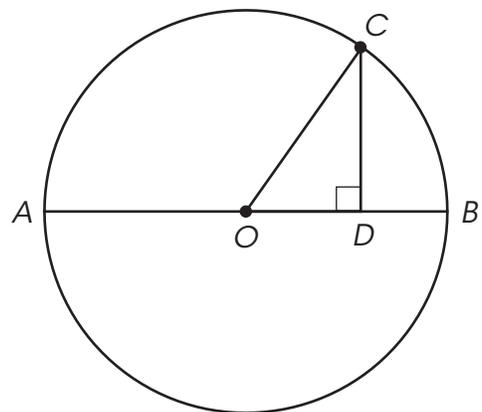
$AO = 6$ cm

$DB = 3$ cm

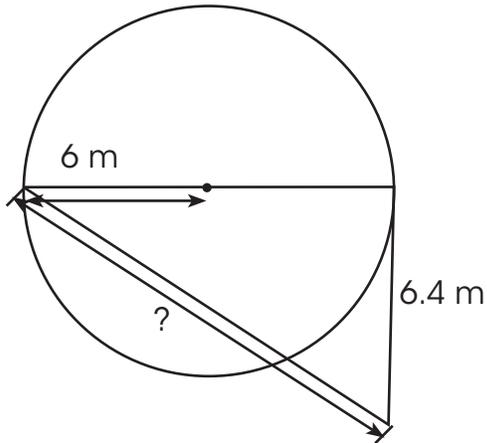
a. What is the length of OC ?

b. What is the length of OD ?

c. What is the length of CD ?



6. Find the missing length.



7. Line segment OS bisects the chord QR . Q is 42° . O is the centre of the circle. What is the value of R ? What is the value of x ?

